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PAWNS AND QUEENS REVISITED: PUBLIC PROVISION OF PRIVATE GOODS WHEN INDIVIDUALS MAKE MISTAKES

Jukka Pirttilä*
Sanna Tenhunen**

* Labour Institute for Economic Research
** University of Tampere and FDPE

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ABSTRACT

Abstract: This paper analyses the optimal tax policy and public provision of private goods when individuals differ in two respects: income-earning ability and rationality. Publicly provided goods should be overprovided or subsidised, relative to the decentralised optimum, if society's marginal valuation of them exceeds the individual valuation and if these goods help relax the self-selection constraints, formulated in a new way. Optimal marginal income tax rates are shown to di¤er from the standard rules if publicly provided goods and labour supply are related.

Keywords: Behavioral economics, optimal taxation, public provision.

TIIVISTELMÄ

Yksityisten hyödykkeiden julkinen tarjonta kun kuluttajat tekevät virheitä

Tässä tutkimuksessa tarkastellaan optimaalista veropolitiikkaa ja julkisten palveluiden tarjontaa kun palveluiden käyttäjät eroavat tuottavuuden ja rationaalisuuden suhteen. Epärationaaliset kuluttajat eivät osaa arvioida julkisten palveluiden arvoa oikein, vaan arvostavat niitä yhteiskunnan näkökulmasta liian vähän. Julkisia palveluja pitäisi tuottaa sitä enemmän, mitä enemmän niiden avulla pystytään pienentämään verotuksen tehokkuustappioita tukemalla työn tarjontaa, ja mitä suurempi on yhteiskunnan ja yksilön julkisten palveluiden arvostuksen ero. Julkisten palveluiden tarjonta vaikuttaa myös optimaalisiin rajaveroihin, kun työn tarjonta riippuu palveluiden käytöstä.

Asiasanat: Psykologinen taloustiede, tuloverotus, julkiset palvelut

1 Introduction

Recent advances in behavioral economics have demonstrated that individual decision-making suffers from bounded rationality and various biases.[1] These biases can be especially problematic in complex decisions that involve uncertainty and dynamism. In these situations, the government might want to intervene, indeed individuals might want the government to intervene, to induce behavior that is closer to what individual wish they were doing. The analysis of such corrective interventions, e.g. through taxes and subsidies, can be called 'behavioral public economics'. In these cases, where the government has an objective function that is different from that of individuals, the government is said to be 'non-welfarist' or paternalistic in its objectives.[2]

One important area where the government could improve upon individual choice is related to goods such as education, health and insurance, which in many countries are indeed often publicly regulated, provided or subsidised. In a recent book, Le Grand (2003) discusses whether the clients of government services should be treated as pawns (that is, recipients whose decisions are mainly delegated to the provider) or queens (sovereign consumers). There are a number of reasons why individuals are particularly prone to make mistakes in decisions in these areas. First, the quantity of information may simply be too great or the causal connections too difficult to understand, relative to the mental capacity of a majority of individuals. Second, mental moods can affect the decisions. Third, especially in the case of education, society might want to make some of the education decisions on behalf of the parents to protect the children's rights. Finally, returns to investments and health often accrue only in the distant future. If individuals have a tendency to undervalue future benefits (e.g. because of hyperbolic discounting, Laibson (1997)), they might be better off if they delegated some of the decision making to an outsider, e.g. the government, to protect themselves against their own

[1] For a survey, see for example Camerer and Loewenstein (2004).

[2] Recent examples of this rapidly expanding field of research include O'Donoghue and Rabin (2003), Sheshinski (2003) and Kanbur, Pirttilä and Tuomala (2004). Seade (1980) provides a seminal analysis.

weakness of will. Treating the customers of public services as pawns instead of queens in certain situations can therefore be desirable.

Our aim is to take these points seriously and consider the optimal public provision of private goods, and the optimal level of subsidies for such goods, when individuals' demand for these goods suffers from the sort of mistakes behavioral economics has highlighted. We examine what will happen to optimal policy if the government tries to correct these mistakes by basing its own decision on what it thinks is truly best for the individuals. In other words, the government's objective function is paternalistic or non-welfarist.

The paper builds on what has become a now standard framework in the literature, that is, the analysis of public provision as a part of the government's redistributive system. In the modern information-based approach to tax analysis, initiated by Mirrlees (1971), there is, by now, a large literature examining the role of publicly provided goods (e.g. Boadway and Keen 1993, Edwards, Keen and Tuomala 1994, Boadway and Marchand 1995, Cremer and Gahvari 1997, and Pirttilä and Tuomala 2002). Because of asymmetric information between the taxpayers and the government, the government must take individuals' incentives into account in its optimal tax policy (technically speaking, through incentive-compatibility constraints). Public provision can then be a useful tool for redistribution if it helps to relax the harmful incentive effects of income taxation.

The present paper differs from the existing literature by assuming that the government's objective function is non-welfaristic. We therefore account for two potential market mistakes: asymmetric information (as in earlier models) and mistakes in individual decision-making. It is interesting to examine whether these two departures from the first best provide a rationale for public provision in similar situations.

Instead of assuming that all individuals make similar mistakes, it is much more realistic to allow, following O'Donoghue and Rabin (2003), for differences in rationality.[3] We therefore examine the case where individuals differ

[3] In the paper, we refer to someone as 'irrational' when his or her preferences differ from what is traditionally seen as rational. Since the government is assumed to be purely rational, an irrational person can, thus, also be described as someone whose preferences differ from the government's preferences.

in two respects, rationality and income-earning abilities (which is clearly needed for the redistribution motive to make sense). Deriving clear-cut results in optimal tax analysis when individuals differ in more than one respect can become notoriously complicated (see, for example the discussion in Boadway, Marchand, Pestieau and Racionero 2002).[4] We therefore follow Blomquist and Christiansen (2003, 2004) and concentrate on a three-type interpretation of the Mirrlees (1971) model. Building on Stern (1982) and Stiglitz (1982), households can be divided into skilled and less-skilled groups. In addition, one of the groups can be either fully rational or partly irrational. Proceeding with the three-type case allows for a much easier intuitive discussion, and yet it fully captures the key mechanisms at work.

Our paper is most closely related to Kanbur, Pirttilä and Tuomala (2004), who examine general non-welfarist optimal tax rules in a continuum case.[5] We simplify the analysis by concentrating on a discrete form of the optimal tax model, and extend the work by considering public provision and differences in rationality. Our study is also related to earlier work, beginning from Musgrave (1959), on merit goods. The optimal tax treatment of merit goods, but not their optimal public provision, is analysed by Sandmo (1983), Besley (1988), Racionero (2001), and Schroyen (2005).[6]

The paper proceeds as follows. To highlight the main intuition, Section 2 presents a benchmark model where all individuals are assumed to be similarly irrational. Section 3 and 4 consider the three-type version in a pooling and separating equilibrium, respectively. Section 5 covers another policy tool, namely using subsidies instead of public provision to affect commodity demand. Section 6 concludes.

[4] Tarkiainen and Tuomala (1999) present simulation solutions for the two-dimensional case.

[5] After finishing the first version of the paper, we learnt of the work by Blomquist and Micheletto (2005) who also consider – as Kanbur et al (2004) – non-welfarist income taxation and the taxation of merit goods. However, they consider neither the public provision of private goods nor two-dimensional heterogeneity.

[6] From these, Racionero (2001) also examines the taxation of merit goods in a mixed tax framework. In contrast to our paper, she considers a case where preferences are additively separable between the merit good and other consumption and between consumption and leisure.

2 The basic model: All irrational

Consider a Stiglitz (1982) -type model with public provision of private goods along the lines of Boadway and Marchand (1995). There are two types of households, 1 and 2. The wage rates of the households are $w_1 < w_2$. The households supply labour l, and their gross income is $y = wl$. The households' skill levels are private information, and the government must design a tax schedule based on observable income instead. The after-tax income of a household is given by $x = y - T(y)$, where $T(y)$ is a non-linear tax schedule set by the government. The household can spend its after-tax income on two goods, a normal consumption good, c, and on another good, e, which is also provided by the government. The extent of government provision is denoted by g; the overall amount available to the household is $e + g \equiv z$. In other words, the households can top up the publicly provided good through their own purchases, e. The partially indirect utility functions of the households, for given pre and post tax income and public provision, are denoted by $v(x, y, g)$.

Examples of the publicly provided good can include investment in education, old-age pension, health or insurance. The key point is that households can rationally and without biases select the consumption of purely private goods, x.[7] However, they have difficulties in designing the correct level of investment on the publicly provided goods. This may be due to the complexity of the decision. A particularly interesting case is one where the costs of publicly provided goods are imminent, but the benefits delayed. If households use hyperbolic discounting, they tend to purchase too little of the publicly provided good from the point of view of their period 0 selves and social welfare. Consider, for example, the case where utility (without public provision) is given by $u(c, e, y) = u_1(c, y) + \beta\delta u_2(e, y)$,where δ is a discount factor. With $\beta < 1$, the households tend to undervalue the benefits of e.

Instead of concentrating on a specific example, we follow Seade (1980) and Kanbur et al. (2004) and work with a general paternalistic social wel-

[7] Of course, in a two good model, if e is chosen incorrectly, so is x because of the budget constraint. We refer to the idea that if x was a vector of private goods, the household can choose among them without mistakes.

fare function $P(x, y, g)$, which can, in principle, differ in any direction from individual utility $v(x, y, g)$. Let us denote the marginal rate of substitution between g and x as $MRS^i_{g,x} = \frac{v^i_g}{v^i_x}$. For ease of interpretation, we concentrate right from the beginning on the case where $\overset{P}{MRS^i_{g,x}} \equiv \frac{P^i_g}{P^i_x} > \frac{v^i_g}{v^i_x} = MRS^i_{g,x}$ for both i, so that from the social welfare point of view, individuals undervalue g. This is also the way Schroyen (2005) finds useful in thinking about merit goods. In the example above, the social welfare function would be one without hyperbolic discounting ($\beta = 1$), i.e. $P(c, e, y) = u_1(c, y) + \delta u_2(e, y)$.

Denote the individual marginal rate of substitution between consumption and income by $s = \frac{v_y}{v_x}$ and the social marginal rate of substitution by $s^P = \frac{P_y}{P_x}$. There is no need for the two to be equal. If $P_g > v_g$, and if the labour supply and publicly provided private good were complements, $s^P > s$, and the government would like the individual to supply more labour than he or she would typically decide himself. One interpretation of this case is that education, health and the like improve the income-earning ability and desire of households. On the other hand, one can imagine that irrationality is related to workaholism, i.e. to a tendency to overwork. Then $s^P < s$.[8]

The social planner is assumed to have a utilitarian social welfare function over P^1 and P^2. The constraints are a resource constraint that the tax revenue must equal the costs of public provision for two individuals, $2rg$ (where r is the marginal rate of transformation between c and g), and a self-selection constraint that the high-ability type must not mimic the choice of the low-ability type. Note that while the social welfare depends on P, the self-selection constraint is similar to the standard model and depends on the utility function generating private behaviour, v.

The lagrangean of the optimisation problem is given by

$$L = P^1(x^1, y^1, g) + P^2(x^2, y^2, g) + \lambda \left[v^2(x^2, y^2, g) - v^2(x^1, y^1, g)\right] + \gamma \left[y^1 + y^2 - x^1 - x^2 - 2rg\right] \quad (1)$$

[8] Hamermesh and Slemrod (2004) provide a detailed analysis of workaholism.

and the first-order conditions by

$$P_x^2 + \lambda v_x^2 - \gamma = 0 \tag{2}$$

$$P_y^2 + \lambda v_y^2 + \gamma = 0 \tag{3}$$

$$P_x^1 - \lambda \widehat{v}_x^2 - \gamma = 0 \tag{4}$$

$$P_y^1 - \lambda \widehat{v}_y^2 + \gamma = 0 \tag{5}$$

where $\widehat{v}$ refers to the mimicker (a type 2 representative mimicking the choice of type 1).

2.1 Marginal tax rates

The individuals choose the labour supply by maximising $v(x, y, g)$ subject to the budget constraint $x = y - T(y)$. From this, the marginal tax rate can be expressed as $MTR = T' = \frac{v_y}{v_x} + 1$. The marginal tax rates for both household types can therefore be derived by dividing (3) by (2) and (5) by (4), respectively, and they are given by

$$MTR(y^2) = \frac{P_x^2}{\gamma}\left(s_2 - s_2^P\right) \tag{6}$$

and

$$MTR(y^1) = \frac{P_x^1}{\gamma}\left(s_1 - s_1^P\right) + \frac{\lambda \widehat{v}_x^2}{\gamma}\left(\widehat{s} - s_1\right), \tag{7}$$

These results give rise to the following proposition:

Proposition 1 *If the publicly provided good is a complement (substitute) to the labour supply, the marginal tax rate for the high-skilled households is negative (positive) and the marginal tax rate for the low-skilled household has an ambiguous (positive) sign.*

If the publicly provided good is a complement to labour supply, i.e. $s_1^P > s_1$, the marginal tax rate for the high-skilled individual is negative, as the government wants to boost the labour supply and indirectly induce the individual to consume more of e. A similar effect is present in the rule for the marginal tax rate for the low-skilled (the first term at the right of (7)), but the rule also depends on a comparison between the marginal rate of substitution of a mimicker and a true type 1 household. This comparison can be signed on the basis of the single-crossing condition. Therefore, $\widehat{s} > s_1$ and the last term in (7)) is positive. The overall sign of the marginal tax rate for the low-skilled household remains ambiguous if the publicly provided good is a complement to the labour supply. In the opposite case, the marginal tax rate for type 1 household is positive.

The gist of these results is that a potential connection between the publicly provided good and the labour supply affects the income tax rules as well. Unlike in a standard model (such as Boadway and Marchand 1995), the income tax rules depend here on the decision on public provision. This result is also derived, albeit in a different setting, by Blomquist and Christiansen (2005), where public provision (daycare) is directly proportional to labour supply.

2.2 Optimal public provision rule

Consider next the welfare impacts of public provision. The derivative of (1) with respect to g can be written as follows:

$$\frac{dL}{dg} = P_g^1 + P_g^2 + \lambda v_g^2 - \lambda \widehat{v}_g^2 - \gamma 2r \tag{8}$$

Substitution from (2) and (4), and adding and subtracting $MRS_{g,x}^1$ and $MRS_{g,x}^2$ yields

$$\frac{dL}{dg} = \sum MRS^i_{g,x} - 2r + \left(\overset{P}{MRS}{}^1_{g,x} - MRS^1_{g,x}\right) + \frac{P^2_x}{\gamma}\left(\overset{P}{MRS}{}^2_{g,x} - MRS^2_{g,x}\right) + \frac{\lambda}{\gamma}\widehat{v}^2_x\left(\overset{P}{MRS}{}^1_{g,x} - \widehat{MRS}_{g,x}\right) \quad (9)$$

where the mimicker's marginal rate of substitution is denoted by $\widehat{MRS}_{g,x}$.

When none of the true ability persons is crowded out, i.e. the publicly provided amount does not exceed the amount inviduals wish to buy themselves[9], individuals' maximisation implies that the marginal rate of substitution is equal to the marginal rate of transformation. For two individuals, this means that $\sum MRS_i = 2r$. Therefore, the sign of $\frac{dL}{dg}$ at the second-best optimum is determined by the rest of the terms. The terms $\overset{P}{MRS}{}^1_{g,x} - MRS^1_{g,x}$ and $\overset{P}{MRS}{}^2_{g,x} - MRS^2_{g,x}$ measure the deviations between the social planner's and the individual's marginal rate of substitution. In the case which we focus on, P_g is larger than v_g, and therefore $\overset{P}{MRS}{}^i_{g,x} > MRS^i_{g,x}$.

The sign of the last term at the right of (9) depends on the relative valuation of g between a mimicker and social planner for type 1. In a standard welfarist case (e.g. in Boadway and Marchand 1995), the comparison is between $MRS^1_{g,x}$ and $\widehat{MRS}_{g,x}$. There, if public provision and the labour supply are complements, the labour supply of a mimicker is smaller than that of true low-skilled person, and $MRS^1_{g,x} > \widehat{MRS}_{g,x}$. However, in our case the sign truly depends on the comparison between $\overset{P}{MRS}{}^1_{g,x}$ and $\widehat{MRS}_{g,x}$ If public provision and the labour supply are complements, the social planner wants the true type 1 individual to consume even more of z than he or she otherwise would. Therefore, $\overset{P}{MRS}{}^1_{g,x} > \widehat{MRS}_{g,x}$. In the case where public provision is a substitute for labour supply, $\overset{P}{MRS}{}^1_{g,x}$ can, however, be smaller

[9]For details see Boadway and Marchand (1995). Note, however, that Blomquist and Christiansen (1998) show that it is optimal to have high-income households crowded out. Since the optimal crowding out decision is not the key for our main emphasis, i.e. paternalistic preferences, we abstract from that discussion in what follows.

than $\widehat{MRS}_{g,x}$. The following proposition summarises:

Proposition 2 *When individuals undervalue z from the viewpoint of social welfare, public provision of z is welfare improving if the labour supply and the publicly provided goods are complements. If the labour supply and the publicly provided goods are substitutes, public provision may or may not be welfare improving.*

To gain intuition for the result, remember that we consider two departures from the first-best world: asymmetry of information and irrationality. To correct for the latter, a paternalistic social planner imposes a positive amount of public provision. To deal with the former distortion, earlier literature, cited in the introduction, has shown that public provision can be used to alleviate the harmful incentive effects arising from distortionary income taxation, if public provision and the labour supply are complements.

These two channels interact in the following way. If the labour supply and the publicly provided good are complements, an increase in public provision both decreases the scope of the mistake individuals make when allocating funds between public and private consumption and also reduces the harmful incentive effects from mimicking. On the other hand, if public provision is a substitute for labour supply, the two effects contradict each other and the net impact remains ambiguous.

In policy terms, there can exist important examples of public provision that are useful tools to deal with both asymmetry of information and irrationality. Policies that increase income-earning abilities and labour force participation, but which individuals tend to undervalue, could include investments in education and health care.

Finally, when the labour supply and the publicly provided good are unrelated (preferences are separable between consumption and leisure), public provision can no longer be used as a tool to reduce incentives to mimicking. The terms related to paternalism nevertheless remain in the provision rule even under separable preferences.[10]

[10] This can be seen more clearly when the provision rule is written as $\frac{dL}{dg} = \sum MRS^h_{g,x} -$

3 Differences in rationality

There is, of course, no need to require that all individuals are equally irrational. As in O'Donoghue and Rabin (2003), it is more useful to assume that irrational individuals form only a part of the population. Given that innate rationality is something the social planner cannot directly observe, the question is how public policy should be formulated to account for both the perfectly and the less-than-perfectly rational households.

Combining this extension with the earlier assumption that households differ in their income-earning ability leaves us with four different types of households. Because of the complications of the problem, here we follow Blomquist and Christiansen (2003, 2004) and concentrate on a somewhat simpler three type model. Type 1 individuals with low wage rates are all assumed to be irrational in their choices over publicly provided goods. Part of the high-income earners are assumed to be irrational (type 2) and part fully rational (type 3). For the latter class, $P = v$.[11]

The formulation of self-selection constraints becomes more complicated. It depends, first, on how the distributional preferences of the government hinge on the rationality of households. Second, the ordering of the two high-ability households now depends on how the labour supply is related to the degree of rationality. Therefore, a large number of different cases can emerge. In order not lose tractability, we concentrate on the case which seems most plausible: the social planner wants to redistribute from the rich to the poor, as in a standard income tax model, and the labour supply and the publicly provided good, z, are complements. Then type 3 households, while consuming more of z, supply more labour than type 2 households. Let

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$2r + \left(\widehat{MRS} - MRS^1\right) + \frac{P_x^1}{\gamma}\left(\overset{P}{MRS}{}^1 - \widehat{MRS}\right) + \frac{P_x^2}{\gamma}\left(\overset{P}{MRS}{}^2 - \widehat{MRS}\right)$. If preferences are separable between consumption and leisure, the term $\left(\widehat{MRS} - MRS^1\right)$ vanishes from the rule.

[11] We could also concentrate on the case where some low-income households were rational and others irrational in contrast to all high-income were irrational. Picking one possibility shortens the notation and discussion, without any implication that we would in any way propagate either option.

us also assume that type 1 households will never want to mimic the choice of high income earners. This means that we must consider two potential self-selection constraints: households 3 should not be allowed to mimic type 2 households, and type 2 households should not be allowed to mimic type 1 households.[12]

Two possible equilibria may emerge: a separating equilbrium, where the social planner picks a separate bundle for all households, or a pooling equilibrium, where the planner cannot distinguish between the two lower income types, and there is a common bundle for type 1 and type 2 households. Let us first concentrate on the pooling equilibrium. The separating equilibirum is dealt with in the next section.

In a pooling equilibrium, type 1 and type 2 households receive the same gross and net income. Thus, there is only one binding self-selection constraint: type 3 households should not mimic the common choice of types 1 and 2. The Lagrangean is now given by

$$L = P^1(x^1, y^1, g) + P^2(x^2, y^2, g) + v^3(x^3, y^3, g) \\ + \lambda \left[v^3(x^3, y^3, g) - v^3(x^1, y^1, g) \right] \\ + \gamma \left[\sum_h \left(y^h - x^h \right) - 3rg \right] \quad (10)$$

Remembering that $x^1 = x^2$ and $y^1 = y^2$ the first-order conditions with respect to $x^h$and y^h, $h = 1, 3$ are

$$(1 + \lambda)\, v_x^3 - \gamma = 0 \quad (11)$$

$$(1 + \lambda)\, v_y^3 + \gamma = 0 \quad (12)$$

[12] Even if one changes the assumptions governing the ordering of the self-selection constraints, the nature of the solution remains similar: public provision depends on a mixture of terms capturing irrationality and terms addressing mimicking behaviour.

$$P_x^1 + P_x^2 - \lambda \widehat{v}_x^3 - 2\gamma = 0 \tag{13}$$

$$P_y^1 + P_y^2 - \lambda \widehat{v}_y^3 + 2\gamma = 0 \tag{14}$$

3.1 Marginal tax rates in the pooling case

It is straightforward to see from the first-order conditions (11) and (12) that the marginal tax rate of type 3 is zero. Marginal tax rates for the other types can be derived by adding and subtracting terms v_x^1 and v_y^1 into equations (13) and (14), respectively, and dividing (14) by (13). The marginal tax rates of types 1 and 2 can be written as

$$MTR^1 = MTR^2 = \frac{\lambda \widehat{v}_x^3}{\gamma}(\widehat{s}^3 - s^1) + \frac{P_x^1}{\gamma}(s^1 - s_1^P) + \frac{P_x^2}{\gamma}(s^2 - s_2^P) \tag{15}$$

There are two channels present in the income tax rate of the irrational types: the first term arises from the self-selection constraint and the last two terms from irrationality. The first term is positive, as $\widehat{v}_x^3 > 0$ and mimicker's marginal rate of substitution is greater than a representative of true type 1 as a result of the single-crossing property. The last two terms depend on the difference between the true type's and the social planner's marginal rates of substitution. When the publicly provided good and the labour supply are complements, $s^P > s$. It is also natural to assume that even in a three-type case, both P_x^1 and P_x^2 are positive. The rule for marginal tax rates in the pooling optimum can thus be stated in the following way

Proposition 3 *The marginal tax rate of the rational high productivity type is zero. The sign of the marginal tax rate of the other two types is ambiguous if the publicly provided good is a complement with the labour supply($s^P > s$), and positive if the publicly provided good is a substitute with the labour supply($s^P < s$).*

Note finally that in a special case where $P_x^1 = P_x^2$, i.e. the government's valuation depends only on income level, not rationality, the rule is the same as in the two-type case.[13]

3.2 Public provision in pooling equilibrium

To find the optimal rule for public provision, we derive the derivative of Lagrangian (10) with respect to g.

$$\frac{dL}{dg} = P_g^1 + P_g^2 + (1+\lambda)\, v_g^3 - \lambda \widehat{v}_g^3 - \gamma 3r \tag{16}$$

After the same kind of manipulation as in the previous section, this can be written as

$$\begin{aligned} \frac{dL}{dg} &= \sum_i MRS^i - 3r \\ &\quad + \left(\overset{P}{MRS}{}^1_{g,x} - MRS^1_{g,x} \right) + \left(\overset{P}{MRS}{}^2_{g,x} - MRS^2_{g,x} \right) \\ &\quad + \left[\frac{P_x^1}{\gamma} - 1 \right] \left(\overset{P}{MRS}{}^1_{g,x} - \widehat{MRS}{}^2_{g,x} \right) + \left[\frac{P_x^2}{\gamma} - 1 \right] \left(\overset{P}{MRS}{}^2_{g,x} - \widehat{MRS}{}^3_{g,x} \right) \end{aligned} \tag{17}$$

The first row again coincides with the individual optimisation when none of the true types is crowded out. In the second row there are two terms arising from irrationality. As the social planner values the consumption of the publicly provided good more than types 1 and 2, these terms are positive.

The terms in the last row arise from mimicking. Consider first the multipliers $\frac{P_x^h}{\gamma} - 1$. Following Blomquist and Christiansen (2004), very useful expressions can be obtained by differentiating the Lagrange function in (10) with respect to x^1 and x^2: $\frac{\partial L}{\partial x^h} = \frac{P_x^h}{\gamma} - 1$, $h = 1, 2$. These expressions represent

[13] This can be seen by combining (13) and (15) to obtain (7).

the welfare effect of a hypothetical increase in net income x^h of household h. If the desired direction of redistribution is from an irrational high ability type towards an irrational low ability type, it can be concluded that $\frac{\partial L}{\partial x^1} > 0$, and $\frac{\partial L}{\partial x^2} < 0$.

To deduce the sign of the terms, we concentrate on a case where g is complement to labour supply. In the first of the terms in the last row of (17), the comparison is between $\overset{P}{MRS}{}^1_{g,x}$ and $\widehat{MRS}{}^3_{g,x}$. If the comparison were between $MRS^1_{g,x}$ and $\widehat{MRS}{}^3_{g,x}$, the sign would not be clear. The true type 1 representative works more, and, since g and the labour supply are complements, tends to favour g more. On the other hand, the type 3 individual is rational and therefore also favours g. Thus, $MRS^1_{g,x}$ can be smaller or larger than $\widehat{MRS}{}^3_{g,x}$. Therefore, it is not clear either whether the government's valuation $\overset{P}{MRS}{}^1_{g,x}$ is smaller or larger than $\widehat{MRS}{}^3_{g,x}$.

The last term in (17) includes a comparison between $\overset{P}{MRS}{}^2_{g,x}$ and $\widehat{MRS}{}^3_{g,x}$. In general, this comparison cannot be signed. However, note that both the mimicker and the true type 2 representative are high skilled, and therefore they supply an equal amount of labour. If the government corrects the valuation of type 2 exactly to the fully rational level, $\overset{P}{MRS}{}^2_{g,x}$ is as large as $\widehat{MRS}{}^3_{g,x}$, and the last term vanishes from the rule.

The following proposition summarises this discussion.

Proposition 4 *Even when consumption of good g is too low from the social welfare point of view and g is complement to the labour supply, public provision of g is not unambiguously welfare improving in a pooling equilibrium.*

When the publicly provided private good and the labour supply are complements, both the aim to reduce the harm from irrationality and asymmetric information *separatively* would speak for a positive level of public provision. However, when these two aims are put together, it is no longer clear that public provision improves welfare both as a tool to correct irrationality and asymmetric information. Taking into account differences in rationality, the two problems, irrationality and asymmetry of information, interact in a way that precludes stating simple policy rules without further assumptions. Note that a result that has a similar character is derived by Jordahl and Micheletto (2005). They consider a model with taste differences and differences in income-earning abilities and show that, in a three-type model, the commodity tax can have an ambiguous sign even if the sign was unambiguous without taste differences.

Finally, it can be noticed that when preferences are separable between the labour supply and the publicly provided private good, in pooling equilibrium we have $MRS^1_{g,x} = MRS^2_{g,x} = \widehat{MRS}^3_{g,x}$. In that case rule (17) reduces to $\frac{dL}{dg} = \sum_i MRS^i - 3r + \frac{P^1_x}{\gamma}\left(\overset{P}{MRS}{}^1_{g,x} - MRS^1_{g,x}\right) + \frac{P^2_x}{\gamma}\left(\overset{P}{MRS}{}^2_{g,x} - MRS^2_{g,x}\right)$. This implies that the social and private optima coincide only when the last two terms are zero. These positive terms would vanish only when the agents are rational, i.e. $\overset{P}{MRS} = MRS$. Thus, irrationality induces a higher level of provision than private optimum would suggest even with separable preferences.

4 Three-type model and a separating equilibrium

The other possible case is a separating equilibrium where each household is given a separate bundle of x and y. Now there are two binding self-selection constraints: type 3 households should prefer his bundle to type 2 household's choice and type 2 households should not want to mimic the choice of type 1 households. The resulting Lagrange function is

$$\begin{aligned} L = {} & P^1(x^1, y^1, g) + P^2(x^2, y^2, g) + v^3(x^3, y^3, g) \\ & + \lambda\left[v^3(x^3, y^3, g) - v^3(x^2, y^2, g)\right] + \delta\left[v^2(x^2, y^2, g) - v^2(x^1, y^1, g)\right] \\ & + \gamma\left[\sum_h \left(y^h - x^h\right) - 3rg\right] \end{aligned} \quad (18)$$

and the first-order conditions are given by

$$v_x^3 + \lambda v_x^3 - \gamma = 0 \tag{19}$$

$$v_y^3 + \lambda v_y^3 + \gamma = 0 \tag{20}$$

$$P_x^2 - \lambda \widehat{v}_x^3 + \delta v_x^2 - \gamma = 0 \tag{21}$$

$$P_y^2 - \lambda \widehat{v}_y^3 + \delta v_y^2 + \gamma = 0 \tag{22}$$

$$P_x^1 - \delta \widehat{v}_x^2 - \gamma = 0 \tag{23}$$

$$P_y^1 - \delta \widehat{v}_y^2 + \gamma = 0 \tag{24}$$

4.1 Marginal tax rates

By standard procedures, the following marginal tax rates can be derived from first-order conditions

$$MTR(y^3) = 0 \tag{25}$$

$$MTR(y^2) = \frac{P_x^2}{\gamma}\left(s_2 - s_2^P\right) + \frac{\lambda \widehat{v}_x^3}{\gamma}\left(\widehat{s}_3 - s_2\right), \tag{26}$$

$$MTR(y^1) = \frac{P_x^1}{\gamma}\left(s_1 - s_1^P\right) + \frac{\lambda \widehat{v}_x^2}{\gamma}\left(\widehat{s}_2 - s_1\right), \tag{27}$$

On the basis of equations (25), (26) and (27), one can write the following proposition:

Proposition 5 *If the publicly provided good is a complement to the labour supply (i.e. $s < s^P$), then the marginal tax rate for the type 3 households is* *zero, and the marginal tax rates for the type 1 and type 2 households have ambiguous signs.*

Note first that since we assume type 3 households are rational, there is no need to distort their choice. The marginal tax rates for the type 1 and 2 households consist of two terms, a corrective term and a standard self-selection term. As in the two type model, these have opposite signs, and the overall signs of the marginal tax rates for 1 and 2 remain ambiguous.

4.2 Public good provision in the separating case

Consider next the welfare impacts of public provision. The derivative of (18) with respect to g can be written as follows:

$$\frac{dL}{dg} = P_g^1 + P_g^2 + v_g^3 + \lambda v_g^3 - \lambda \widehat{v}_g^3 + \delta v_g^2 - \delta \widehat{v}_g^2 - \gamma 3r \tag{28}$$

Following a similar procedure as in earlier sections, equation (28) can be written as

$$\begin{aligned} \frac{dL}{dg} &= \sum MRS^i - 3r \\ &+ \left(\overset{P}{MRS}{}^1_{g,x} - MRS^1_{g,x} \right) + \left[1 - \frac{\delta v_x^2}{\gamma} \right] \left(\overset{P}{MRS}{}^2_{g,x} - MRS^2_{g,x} \right) \\ &+ \frac{\delta \widehat{v}_x^2}{\gamma} \left(\overset{P}{MRS}{}^1_{g,x} - \widehat{MRS}{}^2_{g,x} \right) + \frac{\lambda \widehat{v}_x^3}{\gamma} \left(\overset{P}{MRS}{}^2_{g,x} - \widehat{MRS}{}^3_{g,x} \right) \end{aligned} \tag{29}$$

The interpretation of (29) follows the ones in Sections 2 and 3. The differences $\overset{P}{MRS}{}^h_{g,x} - MRS^h_{g,x}$, $h = 1, 2$, are positive. $\overset{P}{MRS}{}^1_{g,x}$ is likely to be higher than $\widehat{MRS}{}^2_{g,x}$. Both 1 and 2 are irrational and there are therefore no taste differences, whereas the type 1 individual works more. In general, $\overset{P}{MRS}{}^2_{g,x}$ can be smaller or larger than $\widehat{MRS}{}^3_{g,x}$. Following the reasoning in the previous section, they are, however, equal if the social planner corrects the valuation of type 2 exactly to the level chosen by the rational type 3 mimicker. In this case, the last term vanishes from the rule.

The sign of the coefficient $1-\frac{\delta v_x^2}{\gamma}$ is not trivial. Using the first-order condition (21) we find that $1-\frac{\delta v_x^2}{\gamma}=\frac{P_x^2}{\gamma}-\frac{\lambda v_x^3}{\gamma}$. This is negative when $\frac{P_x^2}{\gamma}<\frac{\lambda v_x^3}{\gamma}$. This condition holds when the social planner's valuation of the income of type 2 is sufficiently small. Overall, all terms in (29) are positive or zero, except $\left[1-\frac{\delta v_x^2}{\gamma}\right]\left(\overset{P}{MRS}{}^2_{g,x}-\widehat{MRS}{}^3_{g,x}\right)$. According to this discussion, we can formulate the following proposition.

Proposition 6 *Even when consumption of good g is too low from the social welfare point of view and g is complement to the labour supply, public provision of g is not unambiguously welfare improving in a separating equilibrium.*

The irrationality of type 1 household, implied by the term $\overset{P}{MRS}{}^1_{g,x}-MRS^1_{g,x}$ leads to an unambiguously positive public provision and, thus, the higher the distortion due to irrationality is, the higher the public provision level should be. However, when the term referring to the irrationality of type 2 is negative, it implies that the higher the gap between the socially desirable and the actual level of consumption of type 2 is, the lower the public provision level should be. This can be interpreted so that if government does not value the consumption of type 2 agents enough (i.e. if $\frac{P_x^2}{\gamma}<\frac{\lambda v_x^3}{\gamma}$), their irrationality problem is worsened by public provision. As in the pooling equilibrium, there is again a potential conflict between the two policy goals of the government (correction of asymmetric information and irrationality). Even if there is a straightforward positive linkage between public provision and the labour supply, the use of a single policy instrument is not unambiguously desirable.

The conditions for the optimal provision of the publicly provided good in pooling and separating cases, Equations (17) and (29), look somewhat different. To find the difference between the rules we determine the difference between $\left.\frac{dL}{dg}\right|_{\substack{separating\\equilibrium}}$ and $\left.\frac{dL}{dg}\right|_{\substack{pooling\\equilibrium}}$. After some tedious manipulation there are two terms left:

$$\left.\frac{dL}{dg}\right|_{\substack{separating\\equilibrium}}-\left.\frac{dL}{dg}\right|_{\substack{pooling\\equilibrium}}=\frac{\delta v_x^2}{\gamma}MRS^2_{g,x}-\frac{\delta\widehat{v}_x^2}{\gamma}\widehat{MRS}{}^2_{g,x} \qquad (30)$$

This difference can also be given as $\frac{\delta}{\gamma}\left(v_g^2 - \widehat{v}_g^2\right)$. It is positive (negative), if the true type 2 agent values g more (less) than the mimicker. Because the mimicker works fewer hours than the true type agent, we can conclude that if g and the labour supply are complements, $v_g^2 > \widehat{v}_g^2$ and thus the provision of good g tends to be greater in the separating equilirium than in the pooling equilibrium. Given that in the pooling equilibrium, type 2 households are problematic (as they cannot be separated from poorer type 1 households), it is natural that the difference in (30) hinges on the properties of type 2. In a separating equilibrium, one worries less about this consideration, and perhaps therefore the public provision can more fully account for the needs of type 2 individuals.

Furthermore, it can also be noticed that when preferences are separable between the publicly provided private good and leisure, $v_g^2 = \widehat{v}_g^2$, and the rule for the optimal provision of the publicly provided good is $\frac{dL}{dg} = \sum_i MRS^i - 3r + \frac{P_x^1}{\gamma}\left(\overset{P}{MRS}{}^1_{g,x} - MRS^1_{g,x}\right) + \frac{P_x^2}{\gamma}\left(\overset{P}{MRS}{}^2_{g,x} - MRS^2_{g,x}\right)$, both in the pooling and the separating optima. Therefore, with separable preferences, the socially optimal rule for the publicly provided private good in the separating equilibrium differs from the privately optimal only to the extent that individuals suffer from irrationality.

5 Commodity taxation

Up until now we have concentrated on public provision. As earlier literature has pointed out (starting from Edwards et al., 1994), price subsidies on similar goods can be used as an alternative mechanism to reach the same goals. Here we briefly consider the optimal commodity tax (subsidy) rule in two cases: the benchmark case of two types and the pooling equilibrium in the three type case.[14]

[14] The commodity tax rule in the separating equilibrium is very similar to the simple two type model, and it is therefore presented only in the Appendix.

5.1 Commodity taxation in a two type model

To find the optimal commodity tax we need to redefine our model. Assume now that there is no public provision at all and the agents use all their net income on the consumption of two goods, c and e.[15] We denote the demand by a vector $\mathbf{d}^h = [c, e]$, where the superscript h refers to an agent. Now there is also a price vector $\mathbf{q} = \mathbf{p} + \mathbf{t}$, where vector $\mathbf{p}$ consists of the producer prices and $\mathbf{t} = [0, t]$ is tax levied on good e. Government's and agent's (partially indirect) utility functions are $W(\mathbf{q},x, y)$ and $v(\mathbf{q},x, y)$, respectively. Government's budget constraint is now $\sum(y^i - x^i) + \sum \mathbf{t}\mathbf{d}^i = \overline{g}$, where $\overline{g}$ is given the exogenous revenue requirement (which can also be zero). This constraint can be rewritten on the basis of consumer's budget constraint as $\sum y^i - \sum \mathbf{p}\mathbf{d}^i = \overline{g}$.

Now there are two alternative ways to define how the government's and the individuals preferences differ. First, following Besley (1988), denote the individual demand for e as e^h and the value that the government would like the individual to demand as $e^{W,h}$. Then, since the good in question is a merit good, $e^{W,h} > e^h$. Second, following Schroyen (2005), one can derive the merit goods in terms of how the marginal rate of substitution between e and x, $MRS^h_{q_e,x} = -\frac{V^h_{q_e}}{V^h_x}$, varies with the price, q_e. If the good is a merit good, the marginal rate of substitution is higher for the government than for the individual, i.e. $MRS^{W,h}_{q_e,x} > MRS^h_{q_e,x}$. In this case, an increase in the price of e requires a larger compensation in terms of other consumption for the government than for the individual.[16]

The Lagrangean of the optimisation problem is given by

$$\begin{aligned} L = {} & W^1(\mathbf{q},x^1, y^1) + W^2(\mathbf{q},x^2, y^2) \\ & + \lambda \left[v^2(\mathbf{q},x^2, y^2) - v^2(\mathbf{q},x^1, y^1)\right] + \gamma \left(y^1 + y^2 - \sum \mathbf{p}^T \mathbf{d}^h\right) \end{aligned} \quad (31)$$

[15] The conditions for the case when public provision should be favoured over price subsidies is a complex matter. This question is analysed in a welfarist case by Blomquist and Christiansen (1998).

[16] The latter formulation is similar to one in Blomquist and Micheletto (2005).

and the first-order conditions with respect to x^1, x^2 and $q_e$are now

$$W_x^1 - \lambda \widehat{v}_x^2 - \gamma \mathbf{p}^T \frac{\partial \mathbf{d}^1}{\partial x} = 0 \tag{32}$$

$$W_x^2 + \lambda v_x^2 - \gamma \mathbf{p}^T \frac{\partial \mathbf{d}^2}{\partial x} = 0 \tag{33}$$

$$W_{q_e}^1 + W_{q_e}^2 + \lambda v_{q_e}^2 - \lambda \widehat{v}_{q_e}^2 - \gamma \sum \mathbf{p}^T \frac{\partial \mathbf{d}^h}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial q_e} = 0 \tag{34}$$

We derive the commodity tax rule for both the definitions of merit good, defined above. Note that in deriving the rule in terms of $e^{W,h}$, one utilises Roy's identity to obtain $v_q^h = -v_x^h e^h$. For the government's preferences, such a relation does not necessarily hold, as the arguments in the utility function depend on individual choises. If, however, the government sets its preferences to follow a "modified Roy's rule", i.e. $W_q^h = -W_x^h e^{W,h}$, the comparison is possible. An alternative way is to use marginal rates of substitution $MRS_{q,x}^h$ for the Marshallian demands and the identity given by the Slutsky composition $W_{q_e}^h + W_x^h e^h = W_x^h \left(MRS_{q_e,x}^h - MRS_{q_e,x}^{W,h} \right)$ for the social planner.

The resulting tax rule is, in its two versions [17]

$$t = \frac{\lambda \widehat{v}_x^2}{\gamma S}(e^1 - \widehat{e}^2) + \sum \frac{W_x^h}{\gamma S}(e^{W,h} - e^h) \tag{35}$$

where S is the Slutsky substitution effect $\sum \frac{\partial h_e}{\partial \mathbf{q}}$ and

$$t = \frac{\lambda \widehat{v}_x^2}{\gamma S} \left(MRS_{q_e,x}^1 - \widehat{MRS}_{q_e,x}^2 \right) + \sum \frac{W_x^h}{\gamma S} \left(MRS_{q_e,x}^{W,h} - MRS_{q_e,x}^h \right) \tag{36}$$

The interpretation of this form is similar to the one in which Roy's identity is assumed to hold also for social planner; the marginal rates of substitution is substituted for demands. Thus in the following analysis we concentrate only on the previous form.

The rule for the optimal commodity tax rate in Eq.(35) indicates that

[17] For details see the Appendix.

the sign of the optimal commodity tax depends, first, on the differences in demands between true type 1 agents and mimickers and, second, on the difference between the socially desired and the actual demand. If mimickers consume the good in question less than mimicked agents ($e^1 > \widehat{e}^2$) and consumption is on too low a level from the social point of view ($e^{W,h} > e^h$), a subsidy improves welfare in two ways: it both corrects the rationality problem and makes mimicking less attractive. In the opposite case, where a mimicker consumes more and all irrational households have too high a demand from the social point of view, a commodity tax would improve welfare. However, if terms have opposite signs, i.e. the mimicker is a larger consumer of the good than a true type agent and it would socially beneficial to consume more than is actually done, then the sign of the optimal commodity tax rates remains ambiguous. The rationality problem can be corrected by a negative commodity tax, but at the same time mimicking becomes more appealing. Thus, there is a trade-off between redistribution and rationality objectives. The optimality of commodity taxes can be summarised in the following way.

Proposition 7 *The commodity tax should be negative (positive) if the true type agent consumes more (less) than the mimicker and the desired level of consumption is larger (smaller) than the actual level chosen. Such subsidy (tax) would improve welfare by correcting the consumption distorted by irrationality and by mitigating the self-selection constraint.*

5.2 Commodity taxation in a pooling equilibrium

As in the previous section, here, too, we consider replacing public provision of good g by commodity taxation. Using the same notation as earlier, the Lagrangian can be given as

$$L = W^1(\mathbf{q},x^1,y^1) + W^2(\mathbf{q},x^2,y^2) + v^3(\mathbf{q},x^3,y^3) \\ + \lambda\left[v^3(\mathbf{q},x^3,y^3) - v^3(\mathbf{q},x^1,y^1)\right] + \gamma\left[\sum_h y^h - \sum_h \mathbf{pd}^h\right] \quad (37)$$

The first-order conditions with respect to x^h, $h = 1, 3$ and q_e are needed:

$$W_x^1 + W_x^2 - \lambda \widehat{v}_x^3 - \gamma \sum_{h=1,2} \mathbf{p} \frac{\partial \mathbf{d}^h}{\partial x} = 0 \tag{38}$$

$$(1+\lambda) v_x^3 - \gamma \mathbf{p} \frac{\partial \mathbf{d}^3}{\partial x} = 0 \tag{39}$$

$$W_{q_e}^1 + W_{q_e}^2 + (1+\lambda) v_{q_e}^3 - \lambda \widehat{v}_{q_e}^3 - \gamma \sum_h \mathbf{p} \frac{\partial \mathbf{d}^h}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial q_e} \tag{40}$$

After some manipulations presented in the Appendix, the optimal commodity tax rate can be given[18] as

$$t = \frac{\lambda \widehat{v}_x^3}{\gamma S} \left(e^1 - \widehat{e}^3 \right) + \sum_{1,2} \frac{W_x^h}{\gamma S} \left(e^{W,h} - e^h \right) - \frac{1}{\gamma S} \frac{\partial L}{\partial x^2} (e^1 - e^2) \tag{41}$$

Compared with the two-type economy, the two first terms are alike. The conclusions for the signs of these are similar to the previous section. The last term is analogous to the "redistribution term" received in Blomquist and Christiansen (2004). Since in a pooling equilibrium the government cannot distinguish between type 1 and type 2 individuals, but would like to favour type 1 individuals as they earn less, the government can use the commodity tax as an indirect instrument to affect the welfare levels between the two agents. The derivative $\frac{\partial L}{\partial x^2}$ illustrates the welfare effect of an increase of type 2 agent's net income and it can be assumed to be negative when redistribution is done from high productivity types towards low ability types. With negative S we arrive at the following conclusion.

Proposition 8 *In the pooling equilibrium a subsidy is welfare-improving if*

(i) the true type 1 agent consumes more than the mimicker,

(ii) consumption is at too low a level from the social point of view and

(iii) low skilled type consumes more than high skilled type with the same income.

[18] As in previous section, also here the tax rule can be presented in terms of the marginal rates of substitution by replacing the demands e^h by $MRS_{q_e,x}^h$.

Consider the case where the labour supply and the subsidised good are complements. Then $e^1 > e^2$, as type 1 has lower productivity. However, e^1 can be smaller or larger than $\widehat{e}^3$. The type 1 representative works more and his valuation is therefore high, but the mimicker is rational, and his valuation is also high. In sum, in contrast to the two-type case, when part of the households are rational, it is no longer clear that a subsidy is desirable both from irrationality and asymmetric information reasons, even if the subsidised good is a complement to the labour supply. This result is similar to what was derived in the case of public provision in Section 3.

6 Conclusions

Much of modern governments' activities are not related to night-watchman duties or the provision of pure public goods. A major share of public expenditure is directed to the funding of publicly provided private goods, such as education, health care, care of the elderly, and pension policies. Using a modern, information-based, optimal tax framework, this paper considered two motives for the public provision of private goods: redistribution and paternalism. The latter concern is warranted by recent discussion on behavioural economics indicating that individuals may have a tendency towards biased decisions related to these goods.

Informational asymmetry between the government and the taxpayers implies that incentive effects must be taken into account in redistributive tax policy and the design of public services. Irrational behaviour by individuals provides a potential scope for a paternalistic policy where the government, through its tax and public provision policy, induces behaviour that is closer to what it believes is truly best for the individuals. Our results show that public provision can indeed be used as a mechanism to address these concerns. This is the case, in a model where all households suffer from similar bounded rationality, if the social planner wants to induce individuals consume more publicly provided goods than they themselves would buy and if the publicly provided private good is a complement to the labour supply. Then the paternalistic considerations and a desire to alleviate distortions from income taxation by public provision are aligned.

However, in a richer model where some of the households are fully rational, redistributive and paternalistic objectives can clash even if public provision boosts the labour supply. This result implies that a simple policy tool, public provision, is insufficient to reach both goals in the more general case. The intuition for this somewhat unexpected result stems from the idea that the government may want to value the utility of irrational households less than that of fully-rational households. On the other hand, if redistribution is directed from rich to poor and the fully-rational households work more, the government would like to favour the poorer irrational households. In this case, the two policy objectives point to different directions.

In contrast with standard models of public provision, the decision of public good provision also affects the rules governing the marginal tax rates. This is the case if the labour supply and the demand for publicly provided private good are related. Then the optimal marginal tax rate rules include novel corrective terms.

Subject to the qualifications presented above, it is not entirely implausible that a case exists for public provision of certain goods for paternalistic reasons. Needless to say, the gain from such a policy must be weighed against the potential mistakes the social planner can make when designing paternalistic policies. It is also important to bear in mind the ethical dimensions involved. A paternalistic policy that e.g. protects the individuals is welcomed by some, while others cannot accept such restrictions on individual freedom.

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Appendices

Optimal commodity taxes

A two type case

Starting from the first order condition (34) we apply Roy's identity to the parts $\lambda v^2_{q_e}$ and $\lambda \widehat{v}^2_{q_e}$. Notice that because the optimisation is done with respect to individuals' choices, Roy's identity does not hold for the term $W^h_{q_e}$. Substituting for the first-order conditions (32) and (33) yields

$$W^1_{q_e} + W^2_{q_e} + \left[W^2_x - \gamma \mathbf{p} \frac{\partial \mathbf{d}^2}{\partial x} \right] e^2 + \left[W^1_x - \gamma \mathbf{p} \frac{\partial \mathbf{d}^1}{\partial x} \right] \widehat{e}^2 - \gamma \sum \mathbf{p} \frac{\partial \mathbf{d}^h}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial q_e} = 0 \tag{42}$$

Add and subtract in terms $W^1_x e^1$and $\gamma \mathbf{p} \frac{\partial \mathbf{d}^1}{\partial x} e^1$ to get

$$W^1_{q_e} + W^1_x e^1 + W^2_{q_e} + W^2_x e^2 + \left[W^1_x - \gamma \mathbf{p} \frac{\partial \mathbf{d}^1}{\partial x} \right] \left(\widehat{e}^2 - e^1 \right) - \gamma \sum \mathbf{p} \frac{\partial \mathbf{d}^h}{\partial x} e^h - \gamma \sum \mathbf{p} \frac{\partial \mathbf{d}^h}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial q_e} = 0 \tag{43}$$

Using the properties $\mathbf{p}\frac{\partial \mathbf{d}}{\partial x} = 1 - \mathbf{t}\frac{\partial \mathbf{d}}{\partial x}$ and $\mathbf{p}\frac{\partial \mathbf{d}}{\partial \mathbf{q}} = -\mathbf{d} - \mathbf{t}\frac{\partial \mathbf{d}}{\partial \mathbf{q}}$ (see Edwards et al. 1994), and dropping terms yields:

$$W^1_{q_e} + W^1_x e^1 + W^2_{q_e} + W^2_x e^2 + \lambda \widehat{v}^2_x \left(\widehat{e}^2 - e^1\right) \\ - \gamma \sum \left(1 - \mathbf{t}\frac{\partial \mathbf{d}^h}{\partial x}\right) e^h - \gamma \sum \left(-\mathbf{d}^h - \mathbf{t}\frac{\partial \mathbf{d}^h}{\partial \mathbf{q}}\right) \frac{\partial \mathbf{q}}{\partial q_e} = 0 \quad (44)$$

Notice that $\mathbf{t} = [0, t]$ and $\frac{\partial \mathbf{q}}{\partial q_e} = [0, 1]$, and thus the last row can be written as $-\gamma \sum \left(1 - t\frac{\partial e^h}{\partial x}\right) e^h - \gamma \sum \left(-e^h - t\frac{\partial e^h}{\partial \mathbf{q}}\right) = \gamma \sum t\frac{\partial e^h}{\partial x} e^h + \gamma \sum t\frac{\partial e^h}{\partial \mathbf{q}}$. Finally applying the Slutsky relation $\frac{\partial e}{\partial \mathbf{q}} = \frac{\partial h_e}{\partial \mathbf{q}} - \frac{\partial e}{\partial x} e$, where h_e is the Hicksian demand for e, gives us

$$\sum (W^h_{\mathbf{q}_e} + W^h_x e^h) + \lambda \widehat{v}^2_x (\widehat{e}^2 - e^1) + \gamma \sum t \frac{\partial h_e}{\partial \mathbf{q}} = 0 \quad (45)$$

We denote $\sum \frac{\partial h_e}{\partial \mathbf{q}}$ as S. In the case where only one of the goods is taxed matrix S reduces to scalar number, and thus we can divide the equation by it. In a more general case, where S is a larger matrix, we can multiply the equation by the inverse of S. If one can write $W_{q_e} = -W_x e^P$, a sort of Roy's identity for the social planner, we can rewrite the commodity tax rule as (35).

However, if one wants to avoid the problem with the Roy's identity, denote Marshallian demands by marginal rates of substitution $MRS^h_{q,x} = -\frac{v^h_q}{v^h_x}$ and rewrite the part referring to social planner with help of the Slutsky composition as $W^h_{q_e} + W^h_x e^h = W^h_x \left(MRS^h_{q_e,x} - MRS^{W,h}_{q_e,x}\right)$ to get the form in (36).

Three types and pooling equilibrium

Manipulation is here equivalent to the one in the two type case. From (40) we get

$$W^1_{q_e} + W^2_{q_e} - (1+\lambda)v^3_x e^3 + \lambda \widehat{v}^3_x \widehat{e}^3 - \gamma \sum_h \mathbf{p} \frac{\partial \mathbf{d}^h}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial q_e} = 0 \tag{46}$$

Substituting the first order conditions (38) and (39) into, and adding and subtracting in terms $W^1_x e^1, W^2_x e^2$, $\gamma \mathbf{p} \frac{\partial \mathbf{d}^1}{\partial x} e^1$ and $\gamma \mathbf{p} \frac{\partial \mathbf{d}^2}{\partial x} e^2$ gives

$$\begin{aligned} &W^1_{q_e} + W^1_x e^1 + W^2_{q_e} + W^2_x e^2 \\ &\quad + \left[W^1_x - \gamma \mathbf{p} \frac{\partial \mathbf{d}^1}{\partial x} \right] \left(\widehat{e}^3 - e^1 \right) + \left[W^2_x - \gamma \mathbf{p} \frac{\partial \mathbf{d}^2}{\partial x} \right] \left(\widehat{e}^3 - e^2 \right) \\ &\quad - \gamma \sum_h \mathbf{p} \frac{\partial \mathbf{d}^h}{\partial x} e^h - \gamma \sum_h \mathbf{p} \frac{\partial \mathbf{d}^h}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial q_e} = 0 \end{aligned} \tag{47}$$

Rearranging and adding and subtracting term $\left[W^2_x - \gamma \mathbf{p} \frac{\partial \mathbf{d}^2}{\partial x} \right] e^1$and noticing that $\frac{\partial L}{\partial x^2} = W^2_x - \gamma \mathbf{p} \frac{\partial \mathbf{d}^2}{\partial x}$ yields

$$\begin{aligned} &\sum_{1,2} \left(W^h_{q_e} + W^h_x e^h \right) + \lambda \widehat{v}^3_x \left(\widehat{e}^3 - e^1 \right) + \frac{\partial L}{\partial x^2} \left(e^1 - e^2 \right) \\ &\quad - \gamma \sum_h \mathbf{p} \frac{\partial \mathbf{d}^h}{\partial x} e^h - \gamma \sum_h \mathbf{p} \frac{\partial \mathbf{d}^h}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial q_e} = 0 \end{aligned} \tag{48}$$

Next we use properties $\mathbf{p} \frac{\partial \mathbf{d}}{\partial x} = 1 - \mathbf{t} \frac{\partial \mathbf{d}}{\partial x}$, $\mathbf{p} \frac{\partial \mathbf{d}}{\partial \mathbf{q}} = -\mathbf{d} - \mathbf{t} \frac{\partial \mathbf{d}}{\partial \mathbf{q}}$ and $\frac{\partial e}{\partial \mathbf{q}} = \frac{\partial h_e}{\partial \mathbf{q}} - \frac{\partial e}{\partial x} e$ to rewrite the last two terms, equivalently as in the two type case, as get $\gamma \sum t \frac{\partial h_e}{\partial \mathbf{q}}$. The condition for the optimal commodity tax is thus given by

$$\sum_{1,2} \left(W^h_{q_e} + W^h_x e^h \right) + \lambda \widehat{v}^3_x \left(\widehat{e}^3 - e^1 \right) - \frac{\partial L}{\partial x^2} \left(e^2 - e^1 \right) + \gamma \sum t \frac{\partial h_e}{\partial \mathbf{q}} = 0 \tag{49}$$

Now, when one can write $W_q = -W_x e^P$ and denote $\sum \frac{\partial h_e}{\partial \mathbf{q}}$ as S, the optimal commodity tax rule is given by (41). If the optimal commodity tax rate is preferred to be given in the form of marginal rates of substitution, the rule is given by

$$t = \frac{\lambda \widehat{v}_x^3}{\gamma S}\left(MRS^1_{q_e,x} - \widehat{MRS}^3_{q_e,x}\right) + \sum_{1,2} \frac{W_x^h}{\gamma S}\left(MRS^{W,h}_{q_e,x} - MRS^h_{q_e,x}\right) - \frac{1}{\gamma S}\frac{\partial L}{\partial x^2}\left(MRS^1_{q_e,x} - MRS^2_{q_e,x}\right) \quad (50)$$

Three types and separating case

To consider commodity taxation in separating case we rewrite Lagrange function, following the notation in Section 5, as

$$L = W^1(\mathbf{q}, x^1, y^1) + W^2(\mathbf{q}, x^2, y^2) + v^3(\mathbf{q}, x^3, y^3) + \lambda\left[v^3(\mathbf{q}, x^3, y^3) - v^3(\mathbf{q}, x^2, y^2)\right] + \delta\left[v^2(\mathbf{q}, x^2, y^2) - v^2(\mathbf{q}, x^1, y^1)\right] + \gamma\left[\sum_h y^h - \sum_h \mathbf{p}\mathbf{d}^h\right] \quad (51)$$

To solve the optimal commodity tax rates we need the first-order conditions with respect to x^h, $h = 1, 2, 3$ and q_e:

$$W_x^1 - \delta \widehat{v}_x^2 - \gamma \mathbf{p}\frac{\partial \mathbf{d}^1}{\partial x} = 0 \quad (52)$$

$$W_x^2 - \lambda \widehat{v}_x^3 + \delta v_x^2 - \gamma \mathbf{p}\frac{\partial \mathbf{d}^2}{\partial x} = 0 \quad (53)$$

$$v_x^3 + \lambda v_x^3 - \gamma \mathbf{p}\frac{\partial \mathbf{d}^3}{\partial x} = 0 \quad (54)$$

$$W_{q_e}^1 + W_{q_e}^2 + (1+\lambda)v_{q_e}^3 - \lambda \widehat{v}_{q_e}^3 + \delta v_{q_e}^2 - \delta \widehat{v}_{q_e}^2 - \gamma \sum_h \mathbf{p}\frac{\partial \mathbf{d}^h}{\partial \mathbf{q}}\frac{\partial \mathbf{q}}{\partial q_e} \quad (55)$$

After similar calculations as in the pooling case above, the optimal commodity tax is given by

$$t = \frac{\delta \widehat{v}_x^2}{\gamma S}\left(e^1 - \widehat{e}^2\right) + \frac{\lambda \widehat{v}_x^3}{\gamma S}\left(e^2 - \widehat{e}^3\right) + \sum_{1,2} \frac{W_x^h}{\gamma S}\left(e^P - e^h\right) \tag{56}$$

or alternatively, if the form with $MRS_{q_e,x}^h$ is preferred, the commodity tax rule is

$$t = \sum_{1,2} \frac{W_x^h}{\gamma S}\left(MRS_{q_e,x}^{W,h} - MRS_{q_e,x}^h\right) + \frac{\delta \widehat{v}_x^2}{\gamma S}\left(MRS_{q_e,x}^1 - \widehat{MRS}_{q_e,x}^2\right) + \frac{\lambda \widehat{v}_x^3}{\gamma S}\left(MRS_{q_e,x}^2 - \widehat{MRS}_{q_e,x}^3\right) \tag{57}$$

Now optimal commodity tax in (56) consists of two terms correcting mimicking behaviour and the last term affecting irrationality of type 1 and 2 households. As S is negative, we find that the sign of commodity tax depends on the difference in demand between mimicked and mimicker and between socially desired and actual level of consumption. If the labour supply and the taxed goods are complements, $e^P > e^h$ and $e^1 > \widehat{e}^2$. However, e^2 can be smaller or larger than $\widehat{e}^3$. Therefore a subsidy is not unambiguously welfare improving even in the separating equilibrium.