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RELATIVE WAGES IN MONETARY UNION AND FLOATING

Juhana Vartiainen

PALKAN SAAJIEN TUTKIMUS LAITOS

LABOUR INSTITUTE FOR ECONOMIC RESEARCH
Abstract
We analyse a small open economy with a tradable and a sheltered sector. If the unions that operate in each sector coordinate their wage demands sectorwise, the choice of monetary regime – floating cum inflation target vs. EMU – may affect the relative wages and prices of the economy. We show that EMU results in lower prices for tradable goods and lower real wages in the traded sector while opposite results hold for sheltered sector prices and wages. Thus, if large unions behave strategically, the choice of monetary regime has far-reaching structural implications. JEL E24, E42, J31.

1 Introduction
Does the choice of monetary regime affect relative wages and prices in a small open economy? The intuition of most economists suggests that monetary policy can affect structural economic relationships transitorily at best. However, in corporatist economies like the Nordic and some other European ones, it may turn out that the behaviour of trade unions is affected by the choice of monetary regime. In particular, if it is the case that the trade unions operating in the open sector and the sheltered sector coordinate their action sectorwise, the equilibrium price-wage-relations of the economy may depend on the chosen monetary regime. Such an assumption is not far-fetched in countries like Sweden and Finland where a couple of large unions dominate the scene both in export industries and domestic services. The key assumption is that the unions behave strategically with respect to the economy’s relative prices. These

\footnotesize{\textsuperscript{1}In both Sweden and Finland, for example, unions operating in export industries have discussed the formation of bargaining cartels.}
mechanisms have received only scant attention in the macroeconomic literature, since most models of union bargaining are formulated in a money-neutral way\(^2\). 

## 2 The model

Consider a small open economy with two sectors. One sector produces an internationally traded good or "T-good", whereas the other sector produces a domestic, sheltered good, or "S-good". The workers are organised in both sectors, and, furthermore, all trade unions within a sector coordinate their action and choose a common wage rate. Each union maximises the sum of wages (in their sector), in excess of the reservation wage, taking into account the labour demand function. Let \( p_i \) be the money price of good \( i \) (i = T; S), let \( X_i = X_i(L_i) \) be output as a function of the labour input in the respective sectors, let \( L_i(W_i=p_i) \) denote the labour demand function in terms of the product wage, and let \( W_i \) denote the money wage in sector \( i \). We assume a large number of identical firms but calibrate their number in each sector to unity. The labour demand function is derived in a conventional way: the firms in sector \( i \) maximise their profits \( p_i X_i(L_i) - W_i L_i \), so that \( L_i \) is a function of \( \frac{W_i}{p_i} \). Output in both sectors is then also a function of the sector's product wage; denote \( T \)-sector output by \( F(W_T=p_T) \) and \( S \)-sector output by \( G(W_S=p_S) \), where \( F \) and \( G \) are decreasing functions.

The union objective \( U_i \) in sector \( i \) (i = S; T) is assumed to be

\[
U_i = \frac{W_i}{p_i} \frac{1}{\frac{\partial}{\partial L_i} (L_i/W_i)} - W_r \tag{1}
\]

where \( P \) is the aggregate price index. Expression (1) is the difference between the real (consumption) wage of employed members and the real alternative wage \( W_r \), multiplied by the number of employed workers\(^3\). To keep things simple, we assume that the alternative wage is a given constant in real terms, so that it can be identified with inefficient home production.

Supposing that the unions are strong enough to dictate the wage, maximising (1) yields the following expression for the real wage \( w_i \) in sector \( i \):

\[
w_i = \left( \frac{W_i}{p_i} \right) \left( \frac{\partial}{\partial L_i} (L_i/W_i) \right) - W_r \tag{2}
\]

where \( \frac{\partial}{\partial y} (x) \) denotes the elasticity of variable \( x \) with respect to variable \( y \). This formula, suggested by Rama (1994), summarises the celebrated analysis of Calmfors and Drifill (1986). If a union has no market power at all in the sense that an increase in its own wage cannot be passed to the own product price and the demand for labour is very elastic as well, \( \frac{\partial}{\partial L_i} (L_i/W_i) \) is high in absolute value or even minus infinity. The ratio expression (2) is then unity and the preferred

\(^2\)However, after having worked out the results of this paper, I became aware of the recent paper by Steinar Holden (1999). See the concluding section.

\(^3\)Rama (1994) studies a similar objective function.
wage corresponds to the competitive (alternative) wage $W_r$. This situation corresponds to decentralised firm-level or individual bargaining. On the other hand, a completely encompassing centralised union can to a high extent or completely pass its wage increases to prices so that $\gamma(L_i W_i)$ is low in absolute value and employment does not suffer much from high wages. In that case, however, the effect of wage increases on the price index of the economy is strong or complete so that $\gamma(P_i W_i) = 1$ and higher nominal wages do not increase a member worker’s purchasing power. Even in that case the preferred wage corresponds to the competitive wage. In the intermediate cases, however, expression (2) is above $W_r$ and there is inefficient unemployment.

The value of expression (2) depends on the values of the elasticities $\gamma(L_i W_i)$ and $\gamma(P_i W_i)$. To derive these elasticities, we use the economy’s aggregate goods market equilibrium, which, in such a simple model with no assets, also corresponds to the private sector’s budget constraint (and thereby also to the external balance). Suppose that the consumers consume both goods and their preferences over the two goods can be represented by a Cobb-Douglas formula, so that individual utility is

$$u(x_T; x_S) = x_T^{\sigma_T} x_S^{1 - \sigma_T};$$

where $x_T$ and $x_S$ are the consumed amounts of the $T$-good and $S$-good, respectively. The budget shares out of nominal income are then constant, and we can derive the goods market equilibrium condition

$$p_T F(W_T = p_T) = \hat{\lambda} p_S G(W_S = p_S);$$

where $\hat{\lambda} = \sigma = (1 - \sigma)$. Condition (4) determines the set of possible relative prices once $W_T$ and $W_S$ are determined. Denote the product wage $W_i = p_i$ by $\sigma_i$. Note that $\gamma(L_i W_i)$, the elasticity of employment with respect to the own nominal wage, can be decomposed into two factors, the elasticity of employment with respect to the real wage and (one minus) the elasticity of the own price with respect to the nominal wage, so that

$$\gamma(L_i W_i) = \gamma(L_i \sigma_i)(1 - \gamma(P_i W_i))$$

holds.$^5$

3 Comparison of regimes

We can now derive closed form expressions for the elasticities of formula (2) in different monetary regimes. In monetary union (EMU), the open sector price $p_T$ is given but there is no explicit inflation target, since the European

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$^4$ We will relax this assumption and allow for different elasticities of substitution between the two goods in a later section.

$^5$ Substituting (5) into (2), we get another intuitive expression:

$$w_i \frac{W_i}{W_r} = \frac{\gamma(L_i \sigma_i)}{\gamma(L_i \sigma_i) + \frac{\gamma(P_i W_i)}{1 - \gamma(P_i W_i)}} W_r.$$

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Central Bank does not pay attention to a small country\(^6\). In floating, there is a credible inflation target that ties down the aggregate price index \(P\) but \(p_T\) can fluctuate, being determined by the condition \(p_T = E p_T;F\), where \(p_T;F\) is the foreign currency price of tradables and \(E\) is the exchange rate.

We next derive the open sector and sheltered sector real wage claims in EMU and under floating. In EMU, \(\varepsilon = (L_T W_T) = 0\), since open sector prices are taken as given from the rest of Europe. Using this and (5), the T-wage is, in monetary union,

\[
W_T^{EMU} = \frac{\varepsilon(L_T W_T)}{\varepsilon(L_T W_T) + [1 - \varepsilon(P W_T)] W_T} \quad \text{(6)}
\]

Assume also that the production function of each sector is of the Cobb-Douglas type:

\[
X_T = AL_T^{\alpha_T} ;
\quad (7)
\]

\[
X_{ST} = BL_S^{\alpha_S} ;
\quad (8)
\]

The implied output functions in terms of the product wage are, accordingly,

\[
F(\alpha_T) = \varepsilon_T \frac{\alpha_T}{1 - \varepsilon_T} ;
\quad (9)
\]

\[
G(\alpha_S) = \varepsilon_S \frac{\alpha_S}{1 - \varepsilon_S} ;
\quad (10)
\]

where \(\varepsilon_T\) and \(\varepsilon_S\) are constants. These assumptions imply that the elasticity of employment with respect to the real wage and the elasticity of output with respect to the real wage are constant parameters. The elasticity of the aggregate price index with respect to \(w_T\) can be derived by taking the total differential of the goods market equilibrium condition (4). After some manipulations\(^7\), we get the T-wage

\[
W_T^{EMU} = \frac{\varepsilon(L_T W_T)}{\varepsilon(L_T W_T) + [1 - \varepsilon(P W_T)] W_T} \quad \text{(11)}
\]

\(^6\)Thus, we make the simplifying assumption that all of the economy’s foreign trade is with the euro area.

\(^7\)In the derivation of this formula, we use the fact that \(W_T F^0 = \varepsilon_T (L_T W_T) F^0 (L_T W_T)\) and \(W_S G^0 = \varepsilon_S (L_S W_S) G^0 (L_S W_S)\), where \(F^0(L_T W_T)\) and \(G^0(L_S W_S)\) are the elasticities of output with respect to the labour input. These formulas follow from the definitions of the production and labour demand functions. Dropping the T-S-subscripts for simplicity, we have assumed a production function \(X = X(L)\) which, together with profit maximisation, implies a labour demand function \(L = h(w/p) = h^0\). Furthermore, we can express output as a function of the product wage \(w^p\), so that \(X = F(w^p) = X(h^0)\). Now, by definition, \(\varepsilon(X, L) = X(h^0)\), and, since \(F^0 = X(h^0)\) holds, we have \(F^0 = \varepsilon(X, L)\). Furthermore, \(\varepsilon(L, h^0) = h^0 = h^0\), so that \(F^0 = \varepsilon(X, L) X(h^0) \varepsilon = \varepsilon(X, L) X(h^0)\) and \(F^0 = \varepsilon(X, L) X(h^0)\).
In floating, the domestic currency price of tradables, \( p_T \), is determined by the condition \( p_T = E p_{TF} \), where \( p_{TF} \) is the foreign currency price of tradables and \( p_{TF} \) is the exchange rate. \( E \) is free to fluctuate but the central bank will ensure that a price objective is attained, so that the economy is constrained by an aggregate price constraint.

\[
P (p_S; p_T) = p_T^{E_{pT}} p_S^{1 - \circ} = 1;
\]

(12)

We implicitly assume zero inflation here, but introducing correctly anticipated inflation would not change the results, since the model contains no nominal rigidities. The ideal price index in (12) corresponds to the assumed Cobb-Douglas preferences. Solving \( p_S \) as a function of \( p_T \) from (12), substituting it into (4) and differentiating the latter implicitly yields, after manipulations similar to those above,

\[
W_T^{FLOAT} = \frac{2(L_T^{o_T})}{2(L_T^{o_T}) + 1 - \circ(X_T^{o_T})} W_T; \tag{13}
\]

which differs from (11) only because of the constant \( \circ \) in the last ratio term of the denominator. Comparison of (11) and (13) reveals that

\[
W_T^{EMU} < W_T^{FLOAT} \tag{14}
\]

(note that the denominator and the nominator of the real wage expressions are both negative). Similar computations for the sheltered sector wage reveal that

\[
W_S^{EMU} = \frac{2(L_S^{o_S})}{2(L_S^{o_S}) + 1 - \circ(X_S^{o_S})} W_T; \tag{15}
\]

\[
W_S^{FLOAT} = \frac{2(L_S^{o_S})}{2(L_S^{o_S}) + 1 - \circ(X_S^{o_S})} W_T; \tag{16}
\]

so that

\[
W_S^{EMU} > W_S^{FLOAT}; \tag{17}
\]

Thus, we have shown that the real wage claim of the open sector union is lower in monetary union than in a floating exchange rate cum inflation target regime, whereas the reverse holds for the sheltered sector union. The intuitive explanation is that the open sector can influence its own price in the floating regime: increasing \( W_T \) cuts back demand for the S-sector good and generates an upward pressure for \( p_T = p_S \). This makes it more attractive for the traded sector union to claim a higher wage. For the sheltered sector, there is a reverse situation. In monetary union, it is attractive to claim a high wage since traded goods can be bought at a guaranteed money price but the European central bank does not react to higher domestic costs in the sheltered sector. Thus, EMU lets the sheltered sector pass its costs to prices.
Together, (14) and (17) imply that
\[
\frac{w_T^{\text{FLOAT}}}{w_T^{\text{EMU}}} < \frac{w_T^{\text{EMU}}}{w_T^{\text{EMU}}}
\]
so that the ratio of the sheltered sector wage to the open sector wage will be lower in floating than in EMU. One would accordingly expect EMU to put more pressure on open sector wages.

Furthermore, the relative price ratio \(p_S^T/p_T^T\) is lower in floating than in EMU. Take EMU as the benchmark case. The definition of the price index \(P = p_T^{\text{EMU}}\) implies that \(W_T = p_T^{\text{EMU}}(p_S = p_T)^{1/\gamma}\) and \(W_S = p_S^{\text{EMU}}(p_S = p_T)^{1/\gamma}\). Putting these into (4) and denoting \(p_S = p_T\) by \(\mu\) yields
\[
F\left(w_T^{\text{EMU}}(\mu; \theta)\right) = \mu G\left(w_S^{\text{EMU}}(\mu; \theta)\right);
\]
the RHS of which increases in \(\mu\) while the LHS decreases in \(\mu\). Now, if the economy moves from EMU to floating, \(w_T^{\text{EMU}}\) will be replaced by the higher \(w_T^{\text{FLOAT}}\) and \(w_S^{\text{EMU}}\) will be replaced by the lower \(w_S^{\text{FLOAT}}\) so that the LHS goes down and the RHS goes up, and \(\mu\) must decrease. Thus, as one would expect, the relative price of traded goods is lower in EMU:
\[
\frac{\mu}{p_S^{\text{EMU}}} > \frac{\mu}{p_T^{\text{FLOAT}}}
\]

This results also means that the real exchange rate differs between the regimes. Since \(p_T = E p_T^{T,F}\), higher \(p_T\) due to floating entails higher \(E\) (i.e. weaker domestic currency).

As to employment, a weaker result holds:
\[
\frac{\mu}{p_T^{\text{EMU}}} > \frac{\mu}{p_T^{\text{FLOAT}}}
\]

This statement resembles formulas on comparative advantage; it says that T-employment benefits relatively more (loses less) from EMU membership than S-employment. There seems to be no sharp result available on employment levels, however, since EMU implies both lower \(w_T\) and lower \(p_T\).

We have written the outcome in terms of the real wage claims when each union takes the nominal wage of the other union as given. The setup is of course a bilateral game in which the decision variable of each union is the nominal wage, and the equilibrium analysed above is a Nash equilibrium. Starting from the utility functions (1) and using the goods market equilibrium (4), we can derive the iso-utility surfaces and the implied reaction function of each union in nominal wage space or in real wage space. It turns out that in EMU, the reaction function of the T-union is increasing in \(W_S\) whereas the reaction of the S-union is decreasing in \(W_T\). In floating, the reaction functions are horizontal and vertical.

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8This result will be shown in the next section as part of a more general proposition.
The form of the isoutility curves also suggests that there is a locus of efficient solutions on which the isoutility curves are tangent to each other. The set of efficient solutions deserves a comment. Achieving an efficient solution would presumably require some form of coordinated moderation of wage claims or at least repeated play. Now a potential criticism of the model of this paper is that countries like Finland and Sweden are likely to implement centralised wage bargaining so that the externalities implied by the model do not arise and there need not be any difference in relative wages and prices between the two regimes. However, the relative wage and price comparisons of the previous section can be relevant even in an economy with centralised wage bargaining, since the one-shot Nash equilibrium of the model can plausibly be interpreted as a threat point which will be implemented if centralised negotiations fail. Of course, there are many ways to interpret centralised bargaining in models like this, but one straightforward way to introduce centralisation into our model is to assume that the unions bargain about a pair \((W_T; W_S)\) that lies in the efficient set. It is plausible to think that the unilateral outcome will be implemented if such negotiations fail, so that the two monetary regimes imply different threat points. We do not pursue that analysis formally, since it would be algebraically complicated and deviate somewhat from our main point.

The reaction function formulation also raises the question of stability of the equilibrium. To analyse stability, we have to specify the dynamic structure of the model in a plausible way. The horizontal-vertical reaction functions of the floating regime suggest that the issue of stability hardly arises, since both unions stick to their preferred wage regardless of what the other party does. In the EMU case, the reaction functions turn out to be

\[
W_T = C_T W_S^{K_1}; \tag{22}
\]

\[
W_S = C_S W_T^{K_2}; \tag{23}
\]

where \(C_T\) and \(C_S\) are constants and

\[
K_1 = \frac{\pm(1 \pm)(1 \mp)}{(1 \pm)(1 \pm) + \pm(1 \pm)(1 \pm)} > 0; \tag{24}
\]

\[
K_2 = \frac{\pm(1 \pm)(1 \mp)}{(1 \pm)(1 \pm) + \pm(1 \pm)(1 \pm)} < 0; \tag{25}
\]

To study eventual dynamics, one has to make plausible assumptions about the adjustment process; this is implicitly an assumption about the extensive form of the game. To rule out unstable dynamics, we choose to look at those cases that are potentially most disruptive of stability. Firstly, wage setting may be staggered, so that the reactions alternate. In that scenario, starting from whatever wage-price vector, the T-sector union observes the nominal S-wage
and sets its own wage according to (22). After that, the S-union observes the new T-wage and applies (23); and so on. A second plausible case is simultaneous adjustment: with a given initial wage vector \((W_T; 0; W_S; 0)\), both unions observe the wage of the other party and set their own wage according to (22 and (23); this generates a new vector \((W_T; 1; W_S; 1)\). Both cases generate a sequence of wage vectors and it is straightforward to show that a sufficient condition for convergence is that \(jK_1K_2j < 1\). Inspection of (24) and (25) reveals that to be the case. Consequently, the model is always stable. There may be more cyclical variation in EMU, since the convergence path is a cyclical one in the EMU case.

4 A wider class of consumer preferences

The algebraic expressions above rest on the assumption of Cobb-Douglas consumer preferences. That assumption is restrictive since the elasticity of substitution between the two goods in consumption is thereby set to unity. A wider class of preferences can be accommodated by using a CES formula for consumer utility, namely

\[
u(x_T; x_S) = \left( \frac{x_T^{1/\eta}}{\eta} + \frac{x_S^{1/\eta}}{\eta} \right)^{-\eta}; \tag{26}\]

in which \(\eta > 0\) is the elasticity of substitution between the two goods and \(\eta_T\) and \(\eta_S\) are positive weights. Without loss of generality, we can set \(\eta_T + \eta_S = 1\). The case \(\eta = 1\) corresponds to the Cobb-Douglas case analysed in the previous section. As \(\eta \rightarrow 1\), the two goods become fully substitutable \((u\) becomes linear), whereas the limit case \(\eta = 0\) corresponds to Leontief (fixed coefficients) utility with no substitution at all. With the CES formulation, the ideal price index (dual of utility) is

\[
P = \eta_T p_T^{1/\eta} + \eta_S p_S^{1/\eta}; \tag{27}\]

and the goods market equilibrium condition becomes

\[
(\eta_S = \eta_T)(p_S = p_T) \Rightarrow G(W_S = p_S) = F(W_T = p_T); \tag{28}\]

In the CES case, the real wage formulas (corresponding to (11), (13), (15), (16)) contain the price variables \(p_S\), \(p_T\) and \(P\) and are not by themselves sufficient for inferences on relative prices and the argument becomes more complicated, since we have to analyse equilibria with three variables, both real wages and one price (setting the other price as numéraire).

To keep the mathematics tractable, we assume in this section that the production technology (i.e. the exponent of \(L_i\) in the Cobb-Douglas production function) is the same in both sectors: \(\eta_T = \eta_S\).

With this assumption, it turns out that most of the results of the previous section hold even in the CES economy. We summarise our results on the CES case in the following proposition:
Proposition 1 There is always one unique equilibrium that satisfies the first order conditions of the unions’ maximisation problems and the goods market equilibrium. In that equilibrium, the relative price \( \frac{p_S}{p_T} \) is higher in EMU than in floating. On employment, (21) holds. As to wages, we distinguish two cases: 1) If \( \beta < 1 \), the relative wages always obey the relationship (18)

\[
\text{\mu w}_{\text{EMU}} < \text{\mu w}_{\text{FLOAT}}
\]

2) If \( \beta > 1 \), the open sector real wage is always lower in EMU than in floating and reverse result holds for the sheltered sector real wage; thus, inequalities (14) and (17) continue to hold. Finally, as \( \beta \to 1 \), the difference between the regimes disappears.

These results are proved in the Appendix. The proposition shows that when substitutability is low (\( \beta < 1 \)), the inequality on wages is less sharp; otherwise the results carry over. The difference between the two regimes disappears as the two goods become perfectly substitutable.

5 Comparison with history

We have not touched upon issues of credibility of monetary policy. In the EMU case, it is natural to assume fully credible monetary policy\(^9\), and, we have implicitly assumed that the national central bank’s inflation constraint is fully credible in the floating case. This is in accordance with the constitutional independence of national central banks or the implementation of some kind of trigger strategy.

Another interesting comparison, however, is that between EMU and the devaluation-prone monetary regime that was typical for, say, the Nordic countries under the years of credit rationing. Under that regime, the labour market parties could expect that high nominal wage increases would fully or to some extent be neutralised by letting the exchange rate depreciate. The logical extension of the model of this paper into such an institutional setup is to assume that the large unions of this model take the reactions of the central bank into account in their own decisions. Thus, they not only anticipate changes in the exchange rate but they even try to manipulate that rate.

To fix ideas and relate the model to the Barro-Gordon literature (see Rogoff 1989 for a survey), suppose a Bretton-Woods world in which the central bank can in each period devalue the currency by choosing \( E \) and thereby \( p_T \), since \( p_T = E p_{T,F} \). Furthermore, suppose the central bank has an objective function that values employment but according to which devaluation is costly as well\(^{10}\).

\(^9\)Of course, by saying this I do not mean that the ECB should necessarily be considered as fully credible; but its eventual deviations from its target are not likely to be correlated with the variables of a small open economy.

\(^{10}\)This may be due to the deterioration of reputation as well as the political cost due to conducting negotiations with the International Monetary Fund.
To keep the argument as simple as possible, suppose that the loss function is

$$\text{CB} = \frac{1}{2} \frac{W_T}{p_T} + \frac{1}{2} \frac{W_S}{p_S} + \frac{1}{2} \left(\frac{p_T}{p_T;0}\right)^2;$$

(29)

Thus, the employment objective is captured by the fact that the central bank tries to minimise a weighted average of labour costs ($\frac{1}{2}$; $\frac{1}{2}$ are weights) plus the cost of devaluing the currency ($p_T;0$ is the current traded good price).

The procedure of the game is as follows. The two unions first choose their nominal wage levels and conclude their wage agreements. The central bank then chooses $E$ and thereby $p_T$ and thus determines employment and equilibrium prices.

The reaction function of the central bank can be found by minimising (29); the solution can be expressed by the following equation, which, together with the goods market equilibrium (4) determines prices and real wages:

$$\frac{1}{2} \frac{W_T}{p_T} + \frac{1}{2} \frac{W_S}{p_S} \left(\frac{p_T}{p_T;0}\right)^2 = \left(\frac{p_T}{p_T;0}\right)^2;$$

(30)

In that expression, $\left(\frac{p_T}{p_T;0}\right)^2$ is the elasticity of the sheltered good price with respect to the traded good price that follows from the goods market equilibrium condition when both nominal wages are fixed $^{11}$. Equation (30) can be differentiated (totally) to solve for the elasticity of $p_T$ with respect to both nominal wages; these elasticities embody the central bank's reaction function as far as the unions are concerned:

$$\text{CB} \left(\frac{p_T}{p_T;0}\right) = \frac{1}{2} \frac{W_T}{p_T} + \frac{1}{2} \frac{W_S}{p_S} \left(\frac{p_T}{p_T;0}\right)^2 + 2\left(\frac{p_T}{p_T;0}\right)^2;$$

(31)

$$\text{CB} \left(\frac{p_T}{p_T;0}\right) = \frac{1}{2} \frac{W_S}{p_S} \left(\frac{p_T}{p_T;0}\right)^2 + 2\left(\frac{p_T}{p_T;0}\right)^2;$$

(32)

These expressions contain the wage variables and they cannot be treated as fixed constants; we see, however, that both of these elasticities are positive and belong to the interval $(0;1)$ if the sectors are reasonably symmetric. Thus, both unions will take into account the fact that their wage decision affects the central bank's exchange rate policy according to (31) and (32). The real wage claims of the unions now contain the information on a new channel of influence: instead of the credible inflation target $P = 1$, the unions take into account the above policy elasticities. The real wage claim expressions analogous to (11), (13), (15) and (16) consequently become

$$\text{W_HISTORY} = \frac{2\left(L_T^{0;T}\right)}{2\left(L_T^{0;T}\right) + 1} \left(1 - \frac{\left(\frac{X_T}{X_S} - \frac{X_T^{0;T}}{X_S^{0;T}}\right)}{\left(\frac{X_T}{X_S} - \frac{X_T^{0;T}}{X_S^{0;T}}\right)} \frac{\text{CB} \left(\frac{p_T}{p_T;0}\right)}{\text{CB} \left(\frac{p_T}{p_T;0}\right)} \left(\frac{\text{CB} \left(\frac{p_T}{p_T;0}\right)}{\text{CB} \left(\frac{p_T}{p_T;0}\right)} + 2\left(\frac{p_T}{p_T;0}\right)^2\right)\right) W_T;$$

(33)

$^{11}$Differentiation of the goods market equilibrium yields $\left(\frac{p_T}{p_T;0}\right)^2 = \frac{1}{2}\left(\frac{X_T}{X_S} - \frac{X_T^{0;T}}{X_S^{0;T}}\right)$. 

10
$w_H^{\text{HISTORY}} = \frac{z(L_S^{\circ S})}{z(L_S^{\circ S}) + 1_i \otimes (X_S^{\circ S})_i \otimes CB(p_T W_S)[@(1_i \otimes z(X_S^{\circ S})) + (1_i \otimes (X_T^{\circ T}))]}^{W_T}$

(34)

These expressions resemble the real wage claims of the EMU case given by equations (11) and (15), but both contain an additional term; inspection of these terms shows us that

\[ w_H^{\text{HISTORY}} > w_E^{\text{EMU}} \]

and

\[ w_S^{\text{HISTORY}} < w_S^{\text{EMU}}. \]

Thus, we conclude that EMU membership will dampen the open sector wage claims and increase the sheltered sector wage claims, relative to the old soft monetary policy regime. The soft monetary policy regime made it attractive for the T-sector to drag the central bank into a more devaluationary stance, since a weaker domestic currency affected the two unions asymmetrically: with given nominal wages, devaluations were more harmful for the sheltered sector workers than for the open sector workers. That incentive to manipulate the central bank led to a higher real wage in the open sector. The reverse was true for sheltered sector workers: higher nominal wages would lead the central bank to devalue, which would in turn deteriorate the relative position of the sheltered sector vis-à-vis the open sector.

6 Concluding remarks

We have shown that monetary union membership, together with a corporatist bargaining structure, might not be a neutral choice, as compared to floating with inflation target or the historical experience of soft exchange rate policy. This general point merits further discussion. In most models of the Barro-Gordon literature - and this thinking at present informs the doctrines of European policymakers as well -, the abolition of inflation and the eventual introduction of a common currency is a free lunch and a structurally neutral lunch as well: once credibility is established - be it with a conservative central banker or a long term reputational strategy -, all agents adjust their expectations and real economic variables return to their original levels (while inflation is now lower, which supposedly benefits everybody). This literature treats economic agents as atomistic and there arise no strategic connections; expectations can not be chosen strategically. In corporatist economies with large agents, things may be different.

Whether they are different or not, depends on the institutional setup, however. It is precisely the setup of this paper, namely two (i.e. a modest number of) large unions playing a Nash game, which leads to the non-neutral outcome.
For example, consider what happens if there were a single union only, with an objective function like (1). Consider the following game between the centralised union and the central bank, analogous to that of the previous section: the central union ...rst sets a nominal wage after which the central bank chooses the general price level of the economy. The central union can foresee the bank’s reaction function. One would perhaps expect that such a setup would also imply that introducing credibility would change real variables; however, this is not the case, as can be seen by using (5) to rewrite the real wage claim (2):

\[ w_i = \frac{2(L_i W_i)}{2(L_i W_i) + [1 + 2(P W_i)]W_r} \]

Now, in the central union case \( 2(P W_i) = 2(p W_i) \) holds, and the wage claim reduces to \( \frac{2(L_i W_i)}{2(L_i W_i) + [1 + 2(p W_i)]W_r} \), which is completely independent of whether monetary policy is credible. Thus, if wage bargaining is completely centralised, introducing credibility does not change the economy’s real wage claim. In the soft regime, the central union foresees the central bank’s reaction function and manipulates the price level accordingly; with credible monetary policy, it learns to believe in the announced price level; but the real wage turns out to be the same in both worlds. The same is true, of course, of decentralised bargaining in which no agent is large enough to manipulate the central bank. It is the intermediate case treated in this paper that is interesting.

Our results rest on the simplifying assumptions of monopoly unions. However, it is not obvious that the introduction of Nash bargaining would reverse or neutralise our conclusions. For example, if one assumes that wages are set in a Nash bargain between a sectoral employer federation and the corresponding sectoral union, one can formulate a bargaining problem whereby the union and the employer federation maximise a Nash product \( \frac{\theta(W_i)}{U(W_i)} \), where \( \theta(W_i) \) denotes pro...ts and \( U(W_i) \) is as in (1).\(^{12}\) The rst order condition for this problem, written in elasticity form, is \( \frac{\theta(W_i)}{U(W_i)} + 2(UW_i) = 0 \). Working out the algebra of this expression in our model with Cobb-Douglas consumer preferences, we get the real wage formula

\[ w_i = \frac{W_i}{P} = \frac{1 + 2(L_i W_i)}{1 + 2(L_i W_i) + [1 + 2(P W_i)]W_r}; \quad (38) \]

which resembles (2) and suggests rather similar results.\(^{13}\)

As to practical incomes policy, the result emphasizes the need to keep down sheltered sector costs. Inasmuch as this analysis is applicable to other and larger EU countries, the result also offers a rationale for the tightness of public sector criteria in the Maastricht accord: tight public spending constrains the costs of the sheltered sector.

\(^{12}\)Assuming, of course, that the threat points can be properly calibrated.

\(^{13}\)What the formula also suggests, however, is that some values of \( \theta \) and \( \phi \) may result in corner solutions, since the nominator may be zero for some values of \( \theta(L_i \phi_i) \).
To sum up, we have shown that the choice of monetary regime might not be a structurally neutral choice\textsuperscript{14}. This result rests fundamentally on the fact that EMU integrates monetary policy for economies that are not fully integrated in real terms. It is perhaps plausible to expect that economies will in the long run be more completely integrated so that there will be no genuinely sheltered activity. In the meantime, however, the mechanism of this paper can be relevant for countries contemplating their choice of monetary regime\textsuperscript{15}.

Note. We haven’t included a large survey of related literature into this paper, since the mechanism analysed here has not received much attention in the economic literature (although, of course, many papers by themselves analyse the effects of monetary regimes or the effects of large, strategically acting unions). However, after having worked out the results of this paper, I became aware of Steinar Holden’s recent contribution (Holden 1999). Although Holden’s model is less simplified and contains more variables, the fundamental mechanism of his paper seems to be the same as that of this paper. However, the analysis of the two papers is quite different most of the time, although the results are compatible. Holden’s model is less simplified and perhaps more realistic, while this paper provides a somewhat sharper analytical result on the CES preferences case. Thus, the two papers can be seen as complementary.

References


\textsuperscript{14}Of course, even in this model, the level of inflation does not affect any real variables.

\textsuperscript{15}The EMU case of this paper corresponds formally to the case of an exchange rate target, of course. We have worded this paper in EMU terms, since exchange rate targets have become more theoretical than practical regimes.
A Proof of Proposition 1

Using (2), (27) and (28), it is straightforward to compute the real wage expressions corresponding to equations (11), (13), (15), (16) for the CES case:

\[
\begin{align*}
w_{EM}^T &= \frac{2(L_T^0)^2}{2(L_T^0 T) + \frac{1}{2} \frac{\partial P}{\partial S} \frac{1}{2} \frac{\partial W}{\partial S}} W_r; \\
\end{align*}
\]

\[
\begin{align*}
w_{LO}^T &= \frac{2(L_T^0)^2}{2(L_T^0 T) + \frac{1}{2} \frac{\partial P}{\partial S} \frac{1}{2} \frac{\partial W}{\partial S}} W_r; \\
\end{align*}
\]

\[
\begin{align*}
w_{EM}^S &= \frac{2(L_S^0)^2}{2(L_S^0 S) + \frac{1}{2} \frac{\partial P}{\partial S} \frac{1}{2} \frac{\partial W}{\partial S}} W_r; \\
\end{align*}
\]

\[
\begin{align*}
w_{LO}^S &= \frac{2(L_S^0)^2}{2(L_S^0 S) + \frac{1}{2} \frac{\partial P}{\partial S} \frac{1}{2} \frac{\partial W}{\partial S}} W_r.
\end{align*}
\]

To keep track of regimes, denote the elasticities of the EMU regime by subscript E and those of the floating regime by F. The EMU wage formulas contain the price and wage elasticity terms \(\frac{\partial P}{\partial S}\) (the elasticity of the aggregate price level with respect to the sheltered good price when the traded good price is given), \(\frac{\partial W}{\partial S}\) (the elasticity of the aggregate price level with respect to the sheltered sector nominal wage) and \(\frac{\partial P}{\partial S} \frac{\partial W}{\partial S}\) (the elasticity of the traded good price with respect to the sheltered sector nominal wage). The floating wage formulas contain \(\frac{\partial P}{\partial S} \frac{\partial W}{\partial S}\) (the elasticity of the traded good price with respect to the sheltered good price when the aggregate price level is constant). These elasticities are in general functions of all prices and not given constants. Thus, we have to consider sets of equilibrium equations to compare wages and prices.

In the EMU case, the traded good price \(p_T\) is given and the aggregate price level \(P\) is endogenous. In floating, we have set \(P = 1\) while \(p_T\) and \(p_S\) are endogenous. However, in the EMU case, the real wages are the same, whatever happens to be the nominal level of the foreign good. We can therefore as well compare the floating regime with such an EMU regime that, by coincidence, happens to generate an aggregate price level of unity. Thus, we can set \(P = 1\) even in the EMU case without loss of generality, bearing in mind, of course, that \(p_T^{EMU}\) now becomes algebraically endogenous. With the price level set to unity, we have, in both cases,

\[
\begin{align*}
\frac{1}{2} \frac{\partial P}{\partial S} \frac{1}{2} \frac{\partial W}{\partial S} &= 1; \\
\end{align*}
\]

\[
\begin{align*}
\text{The precise expressions for the price-wage elasticities can be derived from the goods market equilibrium and the definition of the price index. When equation (43) is used to get rid of } p_T, \text{ they are:}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial P}{\partial S} &= \frac{1}{2} \frac{\partial W}{\partial S};
\end{align*}
\]
\[ z_E (p_S W_S) = \frac{2(X_S g_S)}{\pi (X_S g_S)} i^2; \quad (45) \]
\[ z_E (P W_S) = z_E (P p_S) \cdot z_E (p_S W_S); \quad (46) \]
\[ z_F (p_S \rho_F) = \frac{1}{z_F (\rho_F p_S)} = \frac{1}{i^2 (p_S / \pi)}; \quad (47) \]

Substituting these elasticities into the wage formulas (39), (40), (41) and (42), we get new expressions for the real wage claims. It is easiest, however, to consider the inverse of the real wages; thus, we define and compute the "gap between real wage and alternative wage" variables

\[ \frac{\Delta E}{\Delta F} W_r = 1 + \frac{1}{2(L_T g_T)} i^2 (X_T g_T) \frac{\Delta (p_k^e)^{1/z}}{\pi (X_S g_S)}; \quad (48) \]
\[ \frac{\Delta E}{\Delta F} W_T = 1 + \frac{1}{2(L_T g_T)} i^2 (X_T g_T) \frac{\Delta (p_k^e)^{1/z}}{\pi (X_S g_S)(1 - i \cdot p_k^e)^{1/z}}; \quad (49) \]
\[ \frac{\Delta E}{\Delta F} W_S = 1 + \frac{1}{2(L_S g_S)} i^2 (X_S g_S) \frac{\Delta (p_k^e)^{1/z}}{\pi (X_T g_T)}; \quad (50) \]
\[ \frac{\Delta F}{\Delta E} W_T = 1 + \frac{1}{2(L_T g_T)} i^2 (X_T g_T) \frac{\Delta (p_k^e)^{1/z}}{\pi (X_S g_S)(1 - i \cdot p_k^e)^{1/z}}; \quad (51) \]

Furthermore, we can use the fact that \( 1 + \frac{1}{\pi (L_T g_T)} = \frac{2(X_T g_T)}{\pi (L_T g_T)} = \pm \) and \( 1 + \frac{1}{\pi (L_S g_S)} = \frac{2(X_S g_S)}{\pi (L_S g_S)} = \pm \) (recall we have assumed Cobb-Douglas production functions) and define a set of new variables

\[ \frac{\Delta E}{\Delta F} = 1 + \frac{1}{2(X_S g_S)}; \quad (52) \]
\[ \frac{\Delta E}{\Delta F} = 1 + \frac{1}{2(X_S g_S)(1 - i \cdot p_k^e)^{1/z}}; \quad (53) \]
\[ \frac{\Delta F}{\Delta E} = \frac{1}{2(X_S g_S)}; \quad (54) \]
\[ (55) \]

These equations determine the real wages of both regimes as a function of the sheltered sector price \( p_S \) in the regime in question. This gives two equations for the three unknowns in each regime. A third equation is provided by the goods market equilibrium condition (28), which, by using the Cobb-Douglas production formulas (9) and (10), can be written in the form

\[ (56) \]

where, as above, \( \xi_i = \xi_S = \xi_T \). Thus, we have three equations for both regimes: for EMU, equations (52), (54), (56) determine \( \xi^E_S \), \( \xi^E_T \) and \( p^E_S \); and for floating, equations (53), (55), (56) determine \( \xi^F_S \), \( \xi^F_T \) and \( p^F_S \). Our task is to compare these wage-price vectors.

We now introduce our simplifying assumption \( \xi^S_T = \xi^S_S \). Denoting \( \frac{\partial}{\partial p_S} = \frac{\partial}{\partial p^S_S} \) by \( \delta \) and raising both sides of (56) to the power \( 1 = \delta \), we get the modified version of (56):

\[ (57) \]

Our strategy is to analyse the LHS and the RHS of (57) as functions of \( p_S \) (when, below, we refer to LHS and RHS, we mean the RHS and LHS of (57) unless indicated otherwise). Note that the left hand side (LHS) of equation (57) is always decreasing in \( p_S \). The right hand side (RHS) of the same equation depends on the monetary regime. Consider the RHS as a function of \( p_S \) in each regime. Denote it by \( RE(p_S) \) for EMU and \( RF(p_S) \) for floating. Equilibria are found at those values of \( p_S \) at which the LHS crosses the RHS.

Elementary inspection of \( RE(p_S) \) and \( RF(p_S) \) shows that \( RE(p_S) < RF(p_S) \) always holds. Thus, the RHS of equation (56) is always higher in the floating regime than in EMU.

The right hand side of (57) is

\[ (58) \]

for EMU and

\[ \text{RF} (p_S) = \xi^F_S = \xi^F_T \]

for floating.
Combining (57) and (58), we analyse the equation

\[
\mu_2 i_1 \left( 1 + (1 \cdot z) f(x_{T}^{\omega_T}) \right) \frac{\partial}{\partial s} p_s^{1} \quad \text{and} \quad \frac{\partial}{\partial s} \mu_2 i_2 \left( z^2 s_{0}^{\omega_S} \right) \frac{\partial}{\partial s} p_s^{1} 
\]

(59)

for \( \omega = \text{const} \).

Consider first the case \( \omega < 1 \). Since (43) has to hold with positive prices, \( p_s \) is constrained to the interval \((0; (1 \cdot z) \frac{\partial}{\partial s})\). Inspection of the LHS shows that it is

\[
infinite \text{at } p_s = 0 \text{ and drops to zero at the upper limit } \frac{\partial}{\partial s} \text{ of the price interval.}
\]

Inspection (or differentiation) of \( RE(p_s) \) and \( RF(p_s) \) reveals that both of these functions are strictly increasing in \( p_s \) when \( \omega < 1 \): \( \omega_S \) is then increasing and \( \omega_T \) is decreasing regardless of regime. Both \( RE(p_s) \) and \( RF(p_s) \) are negative at \( p_s = 0 \); furthermore, \( RE(p_s) = 0 \) at \( p_s = (\frac{\partial}{\partial s})^{\omega_T} \text{ (not zero)} \). Thus, they must both cross the LHS once and only once. Since \( RE(p_s) \) lies below \( RF(p_s) \) and both cross a decreasing curve, \( RE(p_s) \) must cross the LHS at a higher value of \( p_s \) than \( RF(p_s) \). Thus, when \( \omega < 1 \), \( p_s^R > p_s^F \) must be true. By the same token \( RF(p_s^F) > RF(p_s^F) \). The last inequality implies \( \frac{\partial}{\partial s} < \frac{\partial}{\partial s} \) as claimed in the proposition.

Suppose next that \( \omega > 1 \). Then \( RE(p_s) \) and \( RF(p_s) \) are decreasing curves. For (43) to hold, \( p_s \) must now be constrained to the interval \((0; (\frac{1}{z}) \frac{\partial}{\partial s}) \). At the lower limit of \( p_s \), the LHS is infinite and it converges to \( 0 \) as \( p_s \) increases infinitely. Both \( RE(p_s) \) and \( RF(p_s) \) take infinite values at the lower limit \( p_s = \frac{\partial}{\partial s} \) and at infinity as well, as is apparent from (58) and (59). Thus, by the mean value theorem and continuity of all the three functions concerned, we know that \( RE(p_s) \) and \( RF(p_s) \) must cross the LHS at least once.

This leaves open the possibility of several crossings. We first rule out this possibility for \( RE(p_s) \) by showing that the negative slope of the LHS is always steeper than the negative slope of \( RE(p_s) \), whenever the two curves cross.

Combining (57) and (58), we analyse the equation

\[
\text{const} \times \mu_2 i_1 \left( 1 + (1 \cdot z) f(x_{T}^{\omega_T}) \right) \frac{\partial}{\partial s} p_s^{1} = \left( \frac{\partial}{\partial s} \mu_2 i_2 \left( z^2 s_{0}^{\omega_S} \right) \frac{\partial}{\partial s} p_s^{1} \right) \quad \text{in which we use the fact that } f(x_{T}^{\omega_T}) = f(x_{S}^{\omega_S}) = i_0 \text{ and we have also subsumed } \frac{\partial}{\partial s} \text{ into the constant at the left hand side.}
\]

We want to show that whenever equation (60) holds, the RHS crosses the LHS from below, i.e. the (negative) slope of the LHS is steeper (higher in absolute value) than the negative slope of the RHS. Note that the LHS of (60) is equal to \( \text{const} \times \mu_2 i_1 \left( 1 + (1 \cdot z) f(x_{T}^{\omega_T}) \right) \frac{\partial}{\partial s} p_s^{1} \). Using this form, we can introduce the change of variables \( p_s^{1} = \cdot \) into (60), to get

\[
\text{const} \times [\cdot i_1 \cdot s] = \left( \frac{\partial}{\partial s} \mu_2 i_2 \left( z^2 s_{0}^{\omega_S} \right) \frac{\partial}{\partial s} p_s^{1} \right) \quad \text{in which we use the fact that } f(x_{T}^{\omega_T}) = f(x_{S}^{\omega_S}) = i_0 \text{ and we have also subsumed } \frac{\partial}{\partial s} \text{ into the constant at the left hand side.}
\]

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where $\bar{A} = (\frac{\partial^2}{\partial x^2}) (\frac{1}{y}) > 0$. Multiplying the denominator and the nominator of the \text{rst term of the RHS by}, adding and subtracting $(\frac{2}{i} - 1) \cdot \frac{\partial}{\partial s}$ to and from the nominator and adding and subtracting $(\frac{2}{i} + o) \cdot \frac{\partial}{\partial s}$ to and from the denominator of that same term, we get

$$\text{const} \cdot \left[i \cdot \frac{\partial}{\partial s} \right] \bar{A} = \frac{\mu}{(\frac{2}{i} - 1) (\frac{1}{s}) + \frac{\partial^2}{\partial s^2} \frac{\partial}{\partial s} + (\frac{2}{i} + o) \cdot \frac{\partial}{\partial s}} \frac{\partial^2 + o}{\partial s^2} \quad (62)$$

This enables a new change of variables, defining $\bigtriangledown = \frac{\partial}{\partial s}$. Note that both of these changes of variables have been monotone and increasing, so that they do not change the steepness condition that we want to investigate. By substituting $\bigtriangledown$ for $\frac{\partial}{\partial s}$, differentiating both sides of (62), and taking their difference, we get, by using (62) once more to factor out a part of both sides,

$$\frac{d}{d\bigtriangledown} \text{LHS} - \frac{d}{d\bigtriangledown} \text{RHS} = (\text{const}) \cdot \left(i \cdot \frac{\partial}{\partial s} \right) \frac{\partial^2 + o}{\partial s^2} \frac{\partial}{\partial s} \left(\frac{2}{i} - 1\right) + \frac{\partial^2}{\partial s^2} \left(\frac{2}{i} + o\right) \frac{\partial}{\partial s} \left(\frac{1}{s} \cdot \frac{\partial}{\partial s}\right),$$

where the constant is positive. We want to show that the inside of the brace is negative, establishing that the LHS is steeper than the RHS. Consider the first and third terms of that brace. They have the same nominator, and it is then enough to show that $(\frac{2}{i} - 1) \cdot \frac{\partial}{\partial s} \bigtriangledown < (\frac{2}{i} + o) \cdot \frac{\partial}{\partial s} \bigtriangledown \cdot (\frac{1}{s} \cdot \frac{\partial}{\partial s} \bigtriangledown) holds. That last condition is equivalent to $\bigtriangledown < \frac{5}{4} \cdot (\frac{1}{s} \cdot \frac{\partial}{\partial s} \bigtriangledown)$, which is identically true, since $\bigtriangledown > 0$ by definition. Thus, we know that whenever $R_{E}(p_s)$ crosses the LHS of (56), it crosses it from below. Hence, it can cross it only once.

Consider next consider the case of floating. To rule out the possibility of $R_{F}(p_s)$ crossing the LHS of (56) more than once, rewrite the system of equations associated with floating, using the definition $\frac{\partial}{\partial s} = \frac{\partial}{\partial s} = \frac{\partial}{\partial s} = \frac{\partial}{\partial s}$:

$$\dot{T} = 1 \cdot (\frac{\partial}{\partial s} s \cdot \frac{\partial}{\partial s} (\frac{1}{s} \cdot \frac{\partial}{\partial s} \bigtriangledown))^i;$$

$$\dot{S} = 1 \cdot (\frac{\partial}{\partial s} s \cdot \frac{\partial}{\partial s} (\frac{1}{s} \cdot \frac{\partial}{\partial s} \bigtriangledown))^i;$$

$$\dot{T} = 1 \cdot (\frac{\partial}{\partial s} s \cdot \frac{\partial}{\partial s} (\frac{1}{s} \cdot \frac{\partial}{\partial s} \bigtriangledown))^i;$$

$$\dot{S} = 1 \cdot (\frac{\partial}{\partial s} s \cdot \frac{\partial}{\partial s} (\frac{1}{s} \cdot \frac{\partial}{\partial s} \bigtriangledown))^i;$$

$^{16}$ Suppose that $f$ and $g$ are continuous and differentiable functions and that they are equal at $x_0$ so that $f(x_0) = g(x_0)$. Suppose further that one of them is steeper at that point, say $f'(x_0) > g'(x_0)$. Suppose we define two new functions by applying a monotone and increasing transformation $h$ to both functions. Then the steepness condition becomes $(d/dx) f(x) = h(f(x)) > (d/dx) g(x) = h(g(x))$, which obviously goes on to hold at $x = x_0$. 

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\[ \text{const} \times p_3 \left( \frac{1}{\cosh^2 \theta + \sinh^2 \theta} \right) = \frac{\xi_S}{\xi_T}. \] (65)

Manipulating (65) as above and introducing the change of variable \( \frac{\theta}{2} = \frac{1}{\cosh^2 \theta + \sinh^2 \theta} \), we get the system

\[ \begin{align*}
\dot{\xi}_T &= 1 \bigg[ \frac{1}{\cosh^2 \theta + \sinh^2 \theta} \bigg], \\
\dot{\xi}_S &= 1 \bigg[ \frac{1}{\cosh^2 \theta + \sinh^2 \theta} \bigg], \\
\text{const:} &\cdot \left( \frac{1}{\cosh^2 \theta + \sinh^2 \theta} \right) = \frac{\xi_S}{\xi_T},
\end{align*} \]

in which \( \frac{\theta}{2} \) varies on the interval \((0; 1)\). We show that this system has only one solution. Substituting the first two equations into the third, we get the equation

\[ \text{const:} \cdot \left( \frac{1}{\cosh^2 \theta + \sinh^2 \theta} \right) = \frac{\xi_S}{\xi_T}. \]

The left hand side of (66) increases from 0 to infinity as \( \frac{\theta}{2} \) increases from 0 to 1 while the RHS increases from \( \frac{1}{\cosh^2 \theta + \sinh^2 \theta} \) to \( \frac{1}{\cosh^2 \theta + \sinh^2 \theta} \). Thus, by continuity, there must be a solution. We show that the LHS is steeper than the RHS whenever there is a crossing. From (66), we have, using (66) to factorise out a part of the derivatives,

\[ \frac{\text{d}}{\text{d} \frac{\theta}{2}} \text{LHS} \bigg| \frac{\text{d}}{\text{d} \frac{\theta}{2}} \text{RHS} = \text{const:} \]

\[ \begin{align*}
\frac{1}{\cosh^2 \theta + \sinh^2 \theta} \bigg[ \frac{1}{\cosh^2 \theta + \sinh^2 \theta} \bigg] &\frac{1}{\cosh^2 \theta + \sinh^2 \theta} \bigg[ \frac{1}{\cosh^2 \theta + \sinh^2 \theta} \bigg] \frac{1}{\cosh^2 \theta + \sinh^2 \theta} \bigg[ \frac{1}{\cosh^2 \theta + \sinh^2 \theta} \bigg],
\end{align*} \]

We have to show that the last expression is positive. Consider the first two terms. Since \( \frac{\theta}{2} \) is constrained to lie between zero and unity, \( \frac{1}{\cosh^2 \theta + \sinh^2 \theta} \) attains its maximum value of 1 at \( \frac{\theta}{2} = 1 \). Hence, the first term is greater or equal to \( 4 \sinh^2 \theta \). The denominator of the second term has an in\( \text{m} \)imum at \( \theta = \pm(1 \frac{1}{2}) = 0 \) and \( \frac{\theta}{2} = 0 \). Thus, the second term is always lower or equal to 1 at \( \frac{\theta}{2} = 1 \). The brace is positive if \( 4 \sinh^2 \theta > 1 \), which is implied by the assumption \( \theta > 1 \). Thus, there is one solution only.

It remains to derive the wage inequalities. We know that in (56), \( \text{RE}(p_3) \) and \( \text{RF}(p_3) \) cross the LHS from below and that \( \text{RE}(p_3) < \text{RF}(p_3) \) is true;
hence, $P_E > P_S$ must be true. Note that with $\gamma > 1$, $\xi_S$ is decreasing in $p_S$ in both regimes. Furthermore, $\xi_E > \xi_S$ for all $p_S$. Therefore,

$$\xi_E(p_E) < \xi_S(p_S) < \xi_E(p_S)$$

is true, which implies $w_E^{MU} > w_E^{L0AT}$. By the same token, note that $\xi_T$ is increasing in $p_S$ in both regimes and that $\xi_T < \xi_S$ for all $p_S$. Therefore,

$$\xi_T(p_E) > \xi_T(p_S) > \xi_T(p_S)$$

holds, implying $w_E^{MU} > w_T^{L0AT}$.

Finally, note that regardless of regime, expressions (52), (53), (54), (55) converge to the same function as $\gamma$ grows to infinity.